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Low-Alpha Storage Ring Design for Steady-State Microbunching to Generate EUV radiation

On behalf of Tsinghua SSMB team

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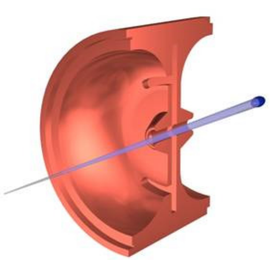
2022/06/16



Accelerator Laboratory of Tsinghua University

Content

- ❑ Introduction
- ❑ Linear lattice design
- ❑ Nonlinear study
- ❑ Summary

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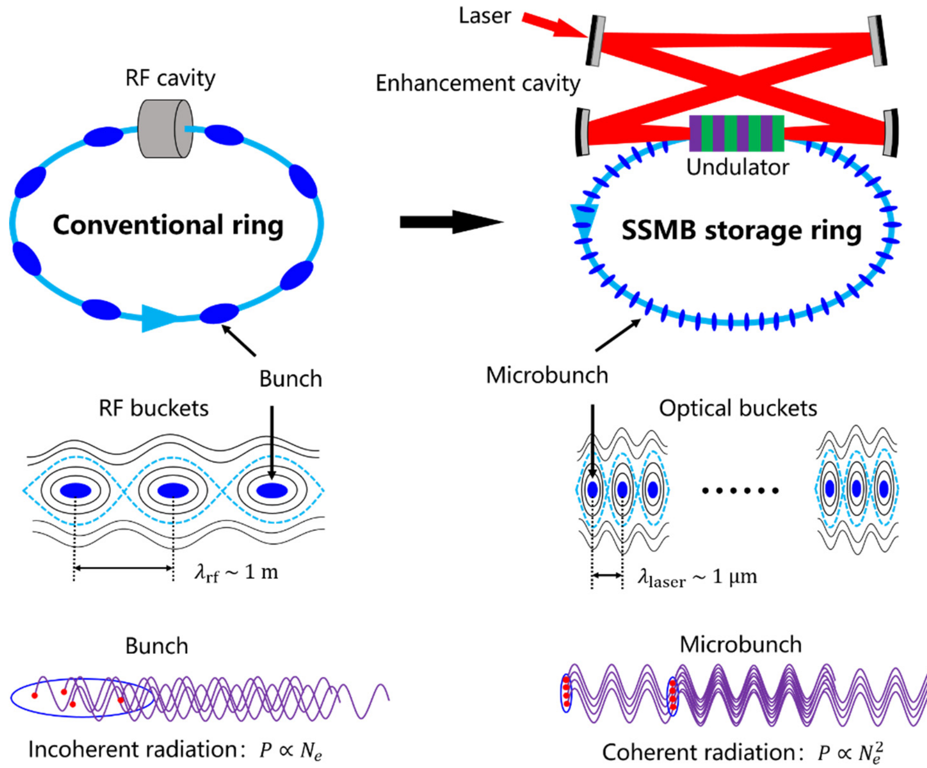
- ❑ **Introduction**
- ❑ Linear lattice design
- ❑ Nonlinear study
- ❑ Summary

Steady-state microbunching (SSMB)*

- **Electron storage ring-based, longitudinal dynamics study needed**
- **Bunching system laser modulator, instead of RF cavity**
- **Two key points**
  - **Microbunching for strong coherent radiation**
  - **turn-by-turn steady state for high repetition rate**
- **High average power, high repetition rate coherent radiation**

6 orders of magnitude extrapolation



* D. F. Ratner and A. W. Chao, Phys. Rev. Lett. 105, 154801 (2010).

* Courtesy of Xiujie Deng

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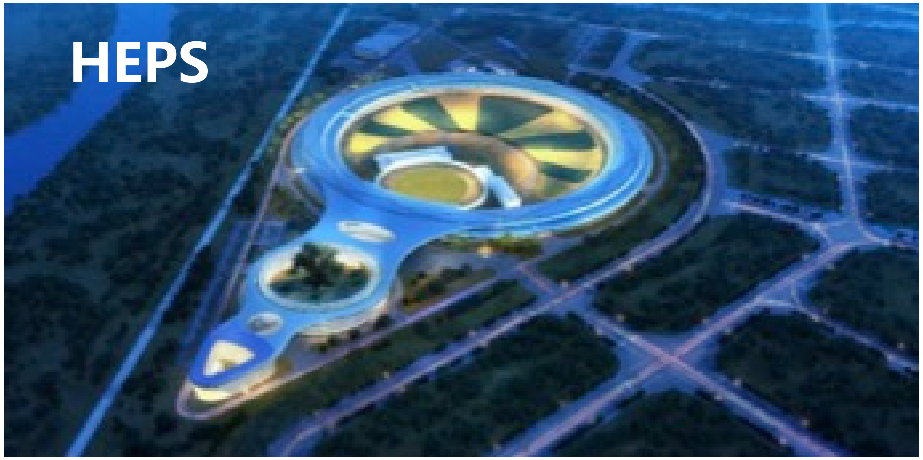
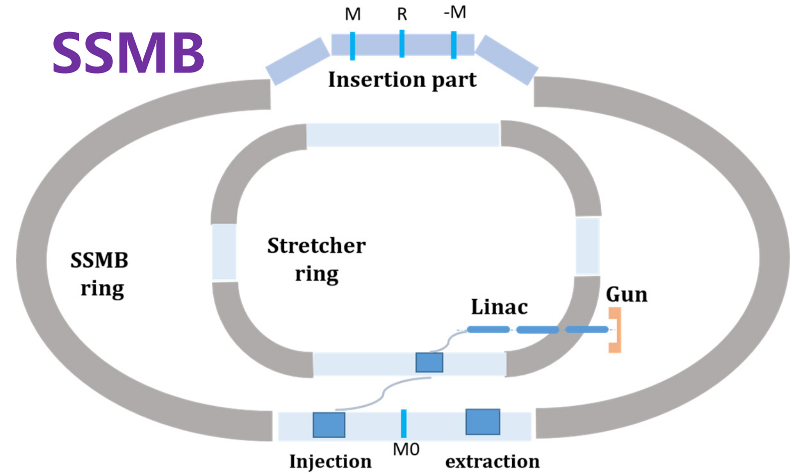
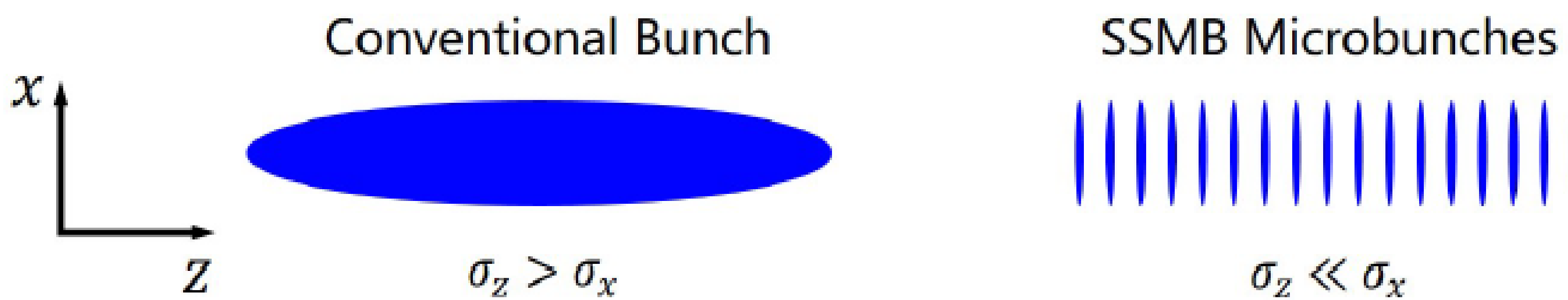
DLSR and SSMB



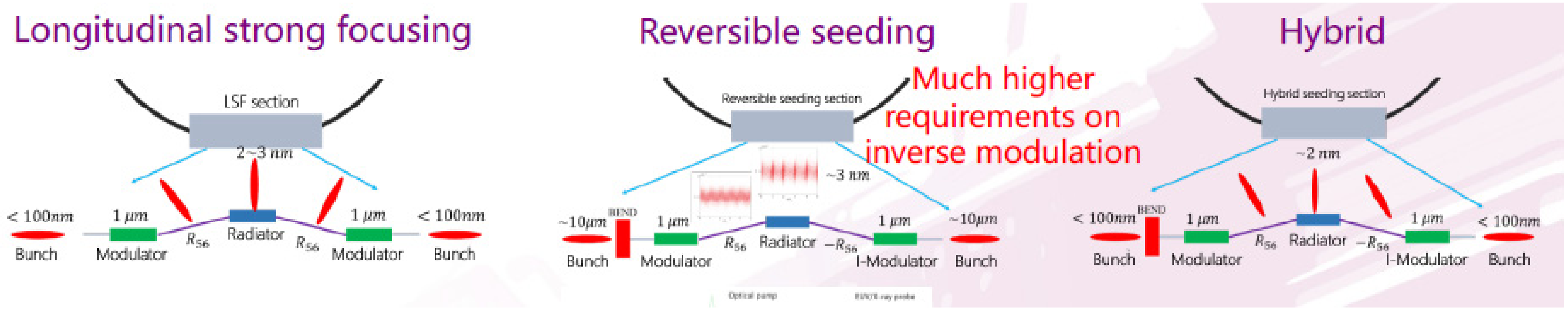




- DLSR: minimize transverse size to diffraction limitation
- SSMB: minimize longitudinal size for coherent radiation

* Courtesy of Xiujie Deng

SSMB schemes



- Low-alpha ring (~100 nm bunch) + LSF(~3nm)
- Required laser power: hundreds MW, pulsed, Duty rate: 1%
- Pulse power : several kW, average power : several tens W

- normal ring + ADM compress (~3nm)
- Required laser power: ~1 MW
- Low bunching factor, coasting beam (@10A)
- Average power : ~ kW

- Low-alpha ring (~100 nm bunch) + ADM compress (~3nm)
- Required laser power: ~1 MW
- high bunching factor
- Average power : ~ kW (@1A)

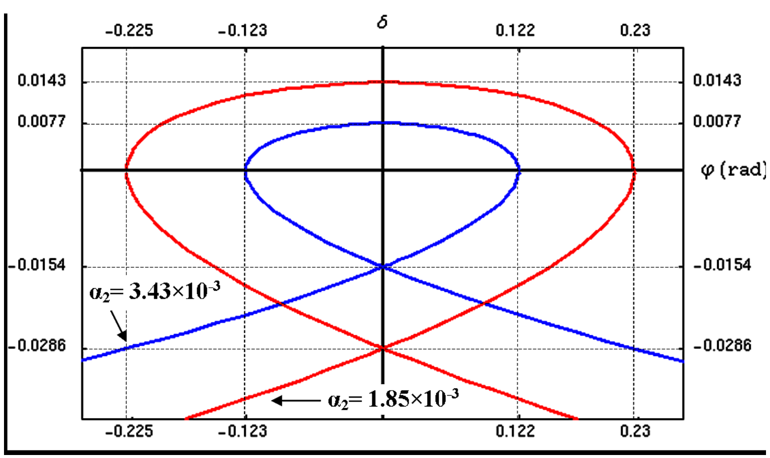
So, what should the SSMB ring looks like? Traditional low-alpha ring or MBA ring? We will find the answer is partially yes but not exactly.

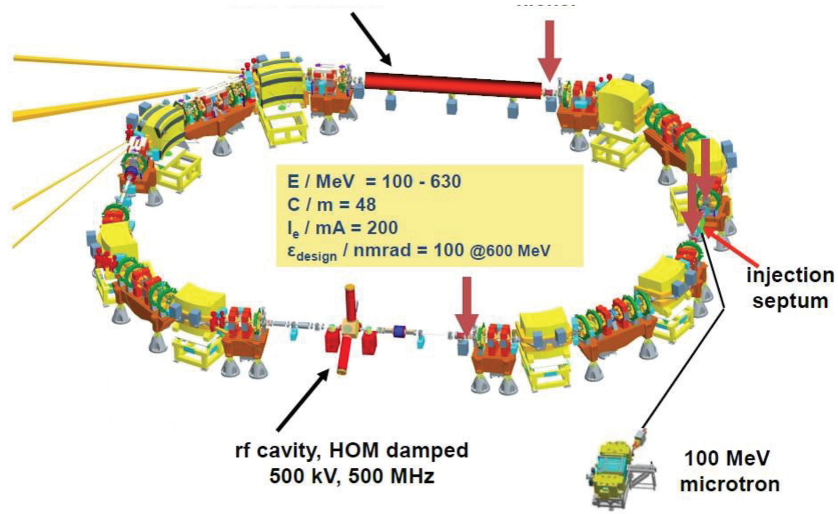
Existing low alpha storage ring

- **Existing low-alpha mode ring**
  - Diamond storage ring:
    $\alpha_1 = -10^{-5}\ or\ -3 * 10^{-6}$
  - SLS storage ring:
    $\alpha_1 = 3.6 * 10^{-5}$
  - MLS storage ring:
    $\alpha_1 = 1.3 * 10^{-4}$

Table 1: Main Parameters of the Two Low Alpha Lattices

Parameter	High ε	Low ε
Emittance	35.2nm.rad	4.4nm.rad
α_1	-3×10^{-6}	-1×10^{-5}
α_2 (no sext.)	0.0116	0.0050
α_3	-0.0426	0.0040
Q_x / Q_y	21.150/12.397	29.390/8.284
Nat. chrom. (ξ_x / ξ_y)	-37 / -26	-66 / -43
$\beta_{x\,ID} / \beta_{y\,ID}$	8.2m / 2.4m	1.1m / 5.7m
Nat. bun. len (3MV)	1.3ps	2.4ps
Synch. freq. (3MV)	346Hz	629Hz





The bunch length in existing low-alpha mode ring:~1ps

I.P.S Martin et al, THPF0371 IPAC'10,Kyoto, Japan
Wustefeld et al TU5RFP005 PAC09
N.P. Abreu et al WE5RFP010 PAC09

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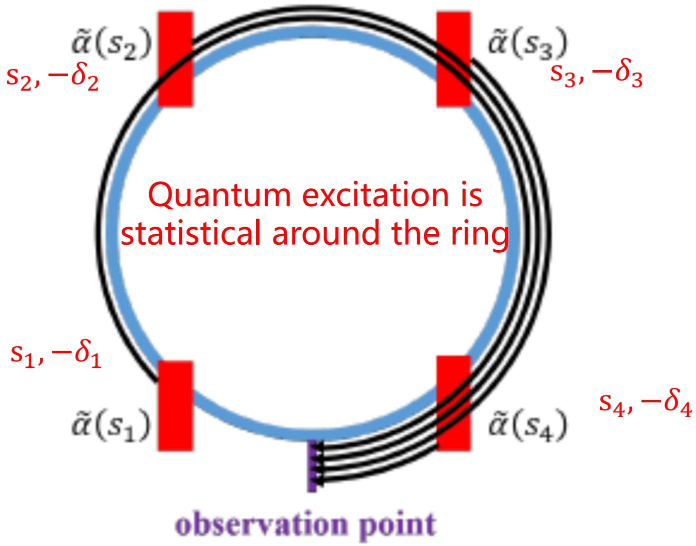
- Introduction
- **Linear lattice design**
- Nonlinear study
- Summary

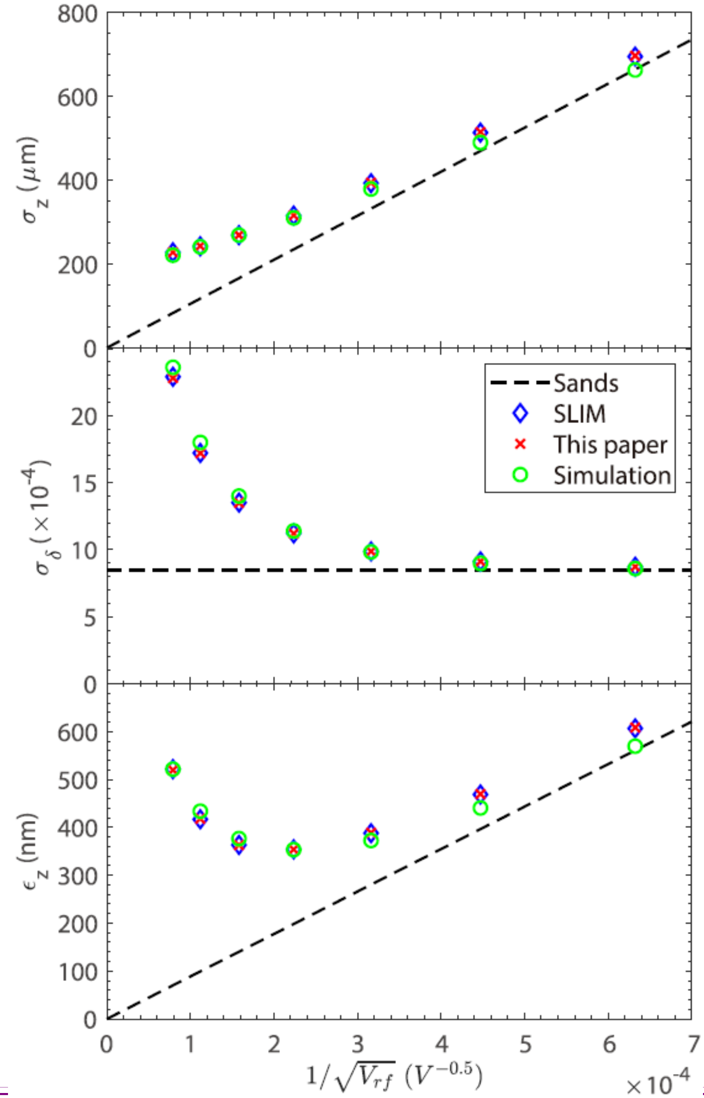
The partial alpha effects



$$\tilde{\alpha}(s_j) = \frac{1}{C_0}\int_{s_j}^{observation\ point} \frac{\eta(s)}{\rho(s)} ds \quad \sigma_\tau = \sigma_\delta \sqrt{(\alpha_c/\omega_s)^2 + T_0^2 I_{\bar{\alpha}}}$$

$I_{\bar{\alpha}}$ the variance of $\tilde{\alpha}(s_j)$

- The scaling law $\sigma_z \propto \sqrt{|\eta|}$ breakdown when $\eta \to 0$
- The key is to control $I_{\bar{\alpha}}$ in low alpha ring

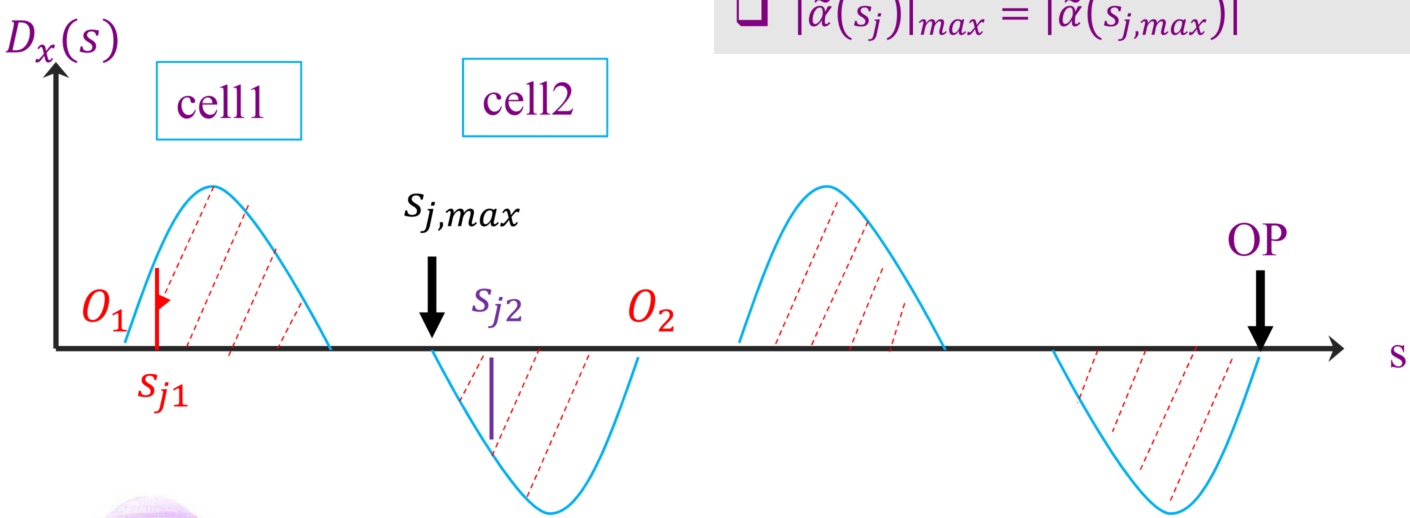


PHD thesis Xiujie Deng [2021]

Partial alpha effects in low alpha ring

- $I_{\tilde{\alpha}} = \left\langle \left(\widetilde{\alpha_{s_j}} - \left\langle \widetilde{\alpha_{s_j}} \right\rangle \right)^2 \right\rangle = \left\langle \widetilde{\alpha_{s_j}}^2 \right\rangle - \left\langle 2\widetilde{\alpha_{s_j}} \right\rangle \left\langle \widetilde{\alpha_{s_j}} \right\rangle + \left\langle \widetilde{\alpha_{s_j}} \right\rangle^2 = \left\langle \widetilde{\alpha_{s_j}}^2 \right\rangle - \left\langle \widetilde{\alpha_{s_j}} \right\rangle^2$
- $\tilde{\alpha}(s_j) = \frac{1}{C_0} \int_{s_j}^{OP} \frac{D_x(s)}{\rho(s)} ds$
- **Control the max of dispersion**
- **The nature idea: make $\alpha_c \sim 0$ in each dipole**

- $R_{56,c} = R_{56,1} + R_{56,2} \sim 0$
- So $\tilde{\alpha}(s_j) < 0$ for all radiation points
- $|\tilde{\alpha}(s_j)|_{min} = 0$
- $|\tilde{\alpha}(s_j)|_{max} = |\tilde{\alpha}(s_{j,max})|$

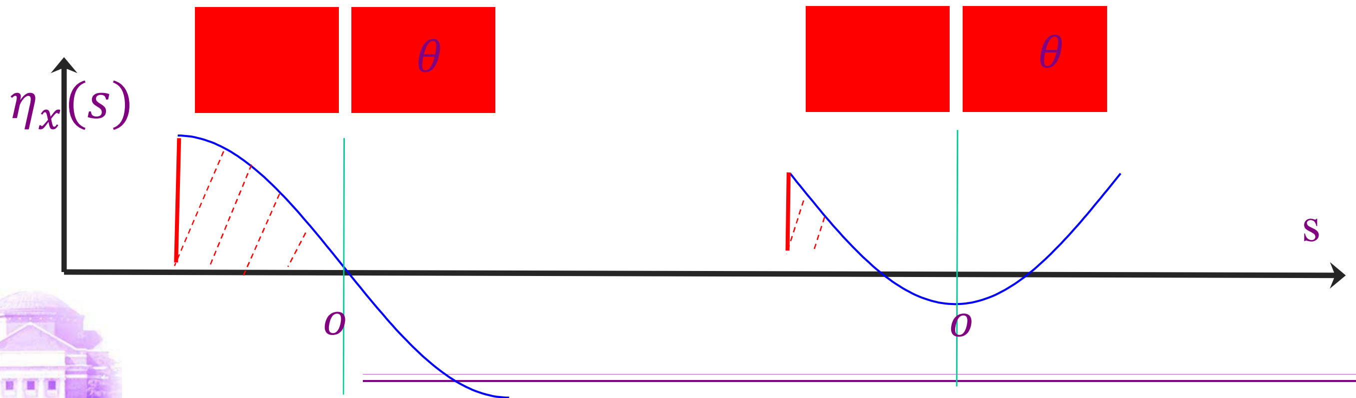


Partial alpha effects in low alpha ring

- In a low alpha cell, $\alpha_c = \frac{1}{C_0}\int_0^L \frac{\eta_x(s)}{\rho(s)} ds \sim 0$

  $\eta(\theta) = \eta_0 cos\theta + \eta_0{}'\rho sin\theta + \rho(1 - cos\theta)$
- Define the dispersion functions at point o as $(\eta_0, \eta_0{}')$
- θ is small, the increment of dispersion in the dipole is

  $\Delta\eta = |\eta(\theta) - \eta_0| = |\eta_0{}'\rho sin\theta + \rho(1 - cos\theta)|$
- bigger the $\Delta\eta$ is, bigger the partial alpha
- The best condition is $\eta_0{}' = 0$



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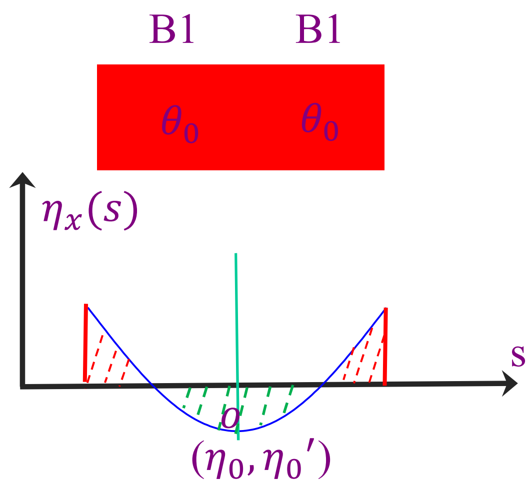
Minimize partial alpha in dipole

- Make the α_c in each dipole be 0
- At the middle of the dipole $\eta_0' = 0$



- $\eta(\theta) = \eta_0 cos\theta + \eta_0' \rho sin\theta + \rho(1 - cos\theta)$
- $\int_0^{\theta_0} \eta(\theta) d\theta = 0$

- We get $\eta_0' = \dfrac{\sin(\theta_0) - \theta_0 - \frac{\eta_0}{\rho}\sin(\theta_0)}{1 - \cos(\theta_0)}$
- And $\eta_0' = 0$, so $\eta_0 = \rho \dfrac{(\sin(\theta_0) - \theta_0)}{\sin(\theta_0)} \approx -\frac{1}{6}\rho{\theta_0}^2$, and $\eta(\theta) = \frac{1}{3}\rho{\theta_0}^2$

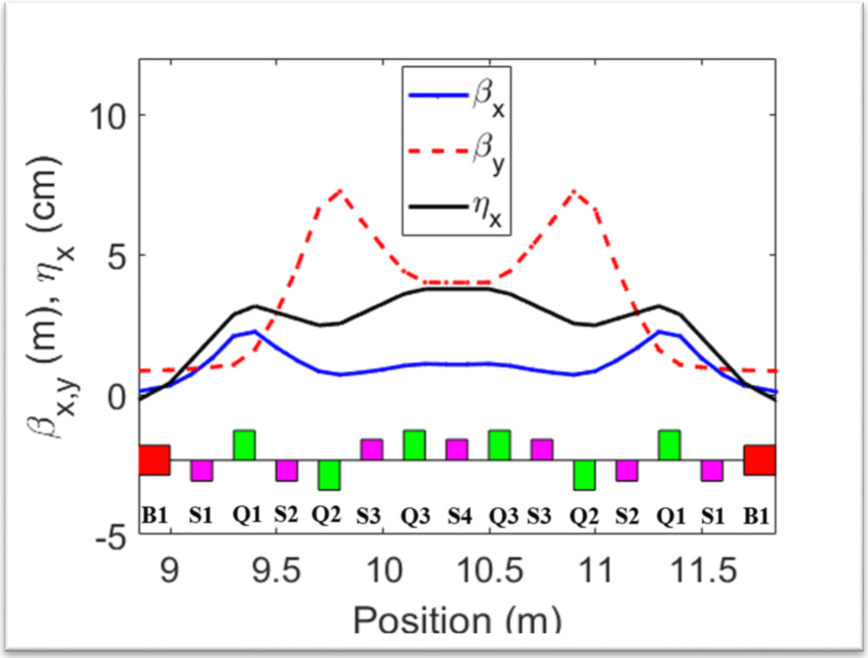
$$\tilde{\alpha}(\theta) = \frac{1}{C_0}\rho\left(\theta - \frac{\theta_0}{\sin\theta_0}\sin\theta\right) \qquad I_{\tilde{\alpha}(s_j)} = \langle\tilde{\alpha}(\theta)^2\rangle - \langle\tilde{\alpha}(\theta)\rangle^2 \approx \frac{\sqrt{2415}}{2520 C_0}\rho\theta_0^3$$

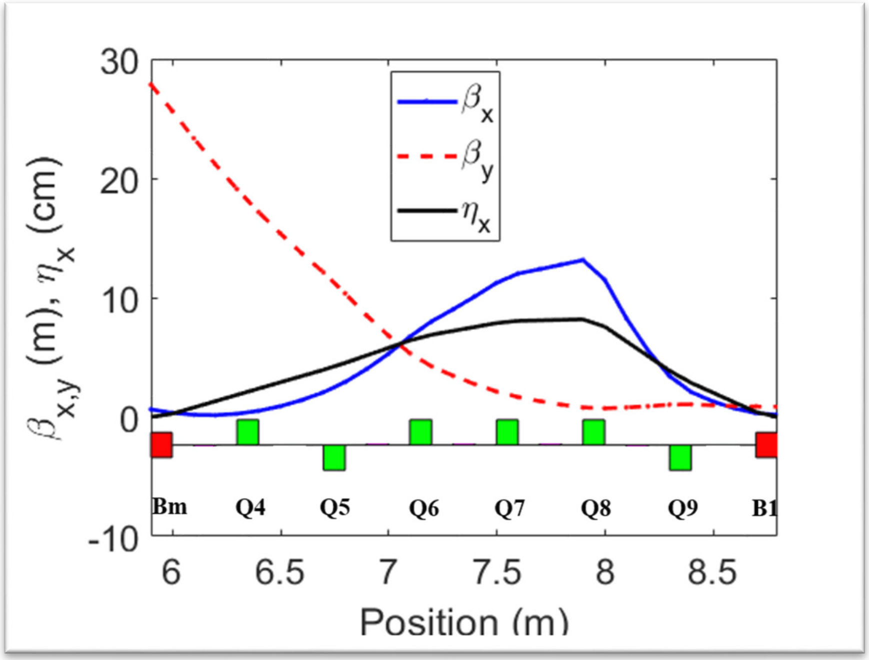
If make $\sigma_{s,partial\ alpha} = C_0\sigma_\delta\sqrt{I_{\tilde{\alpha}(s_j)}} \sim 10\ \text{nm}$ $\quad \rho = 2.0\ \text{m},\ \sigma_\delta = 2.4 \times 10^{-4}$

Then $\theta_0 < 5.86$ deg

Lattice design test

Adjust quadrupole strengths to control ν_x , ν_y, η_0





Main Cell

Tune: 0.75, 0.25

$$\eta_0 = -\frac{1}{6}\rho\theta^2$$

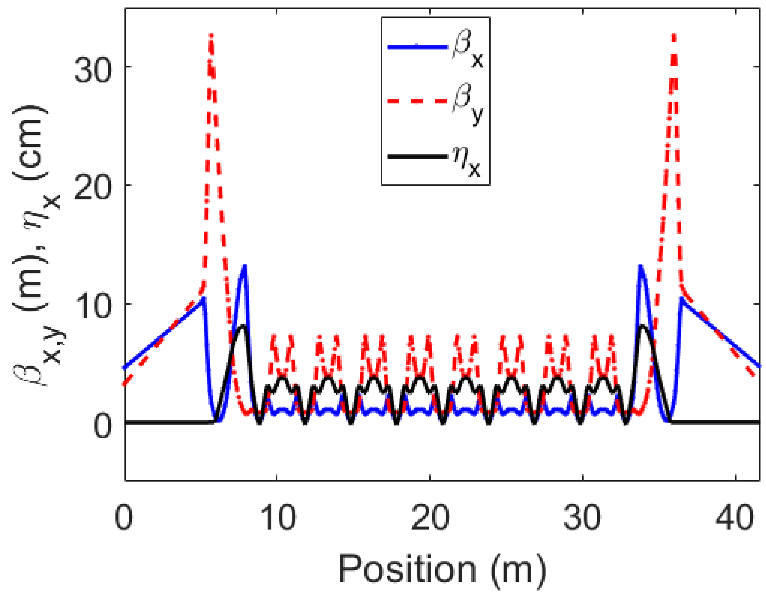
Match Cell

Match the dispersion and alpha for straight

Lattice parameters

E_0 =400 MeV

Parameters	Value
Circumference[m]	166.78
Tunes(x/y)	30.677/11.391
Chromaticity(x/y)	0.006/-0.0008
alphac	2.11e-6
alphac2	6.39e-5
alphac3	7.13e-3
partial alpha	3.07e-7
Sdelta0 σ_δ	2.52e-4
U0 [keV]	1.23
Natural emittance [pm]	54.4
Hor./Ver damping time [ms]	361.4/362.2
Long. Damping time [ms]	181.3



Twiss for super-period

$$\tilde{\alpha}(s_j) = \frac{1}{C_0}\int_{s_j}^{obser} \frac{D_x(s)}{\rho(s)}\, ds - \frac{1}{\gamma^2}\frac{C_0 - s_j}{C_0}$$

$$\sigma_{s,partial\ alpha} = C_0\sigma_\delta\sqrt{I_{\tilde{\alpha}(s_j)}} = 12.83\ nm$$

Single particle tracking for the lattice

- 500 macro particles
- 1.5 million turns (~ 4.5 damping time)
- All particles survived
- Rms bunch length: 40 nm (simulation)

$\lambda = 1\mu m$	
Voltage	Half bucket height δ_{rf}
250 kV	2.22×10^{-3}
Synchrotron tune ν_s	Bunch length $\sigma_{s,sands}$
0.089	36.07 nm

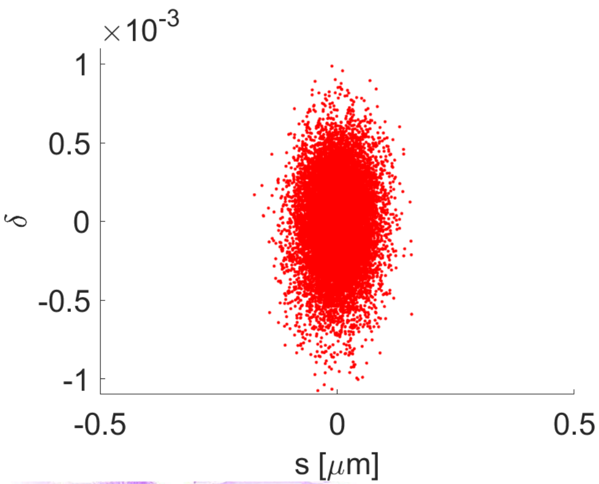
$$\sigma_s = \sqrt{\sigma_{s,sands}^2 + \sigma_{s,partial\ alpha}} = \sqrt{12.83^2 + 36.07^2} = 38.29\ nm.$$

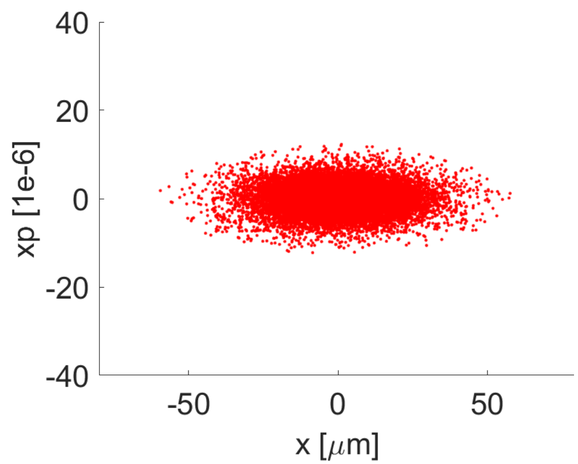
$$V_0[V] = \sqrt{2}E_0 R_z J k_u \omega_0 \int_0^{\frac{L_u}{2R_z}} \frac{A}{\sqrt{1+z^2}} e^{-\frac{A^2}{1+z^2}} dz$$

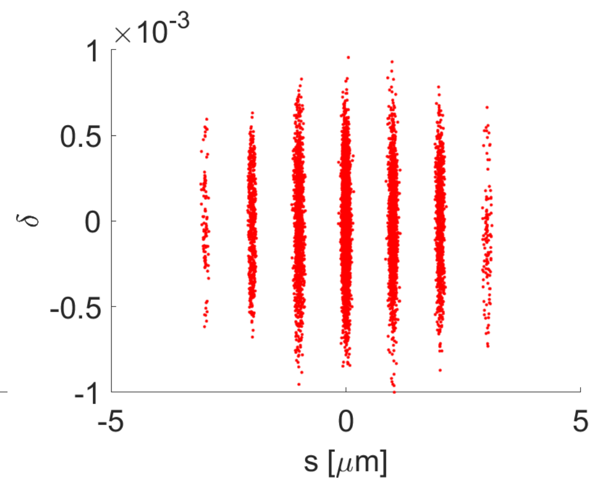
$$P[W] = \frac{\pi \omega_0^2}{4} E_0^2 \sqrt{\frac{\varepsilon_0}{\mu_0}},$$

L_u =2.5 m, $\omega_0 = 450\ \mu m$
$\lambda = 1\ \mu m$, N=500, K=6.86

V_0: 250 kV→ P: 0.56 MW







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The difference between SSMB ring and MBA ring

- ❑SSMB rings focus on longitudinal dynamics, minimizing longitudinal emittance will naturally require the MBA structure.
- ❑A MBA structure will not naturally be the SSMB ring, there is precise dispersion control in dipole for SSMB rings.
- ❑SSMB rings do not require very low transverse emittance, instead prefer relatively larger transverse emittance considering IBS effects

Content

High order momentum compaction factor

☐ Second order alpha

- Destroy RF bucket if too large

$$\alpha = \alpha_c + \alpha_2\delta + \cdots$$

$$\alpha_{c2} = \frac{1}{C}\int_0^C \left(\frac{\eta_2}{\rho} + \frac{{\eta_1'}^2}{2}\right) ds$$

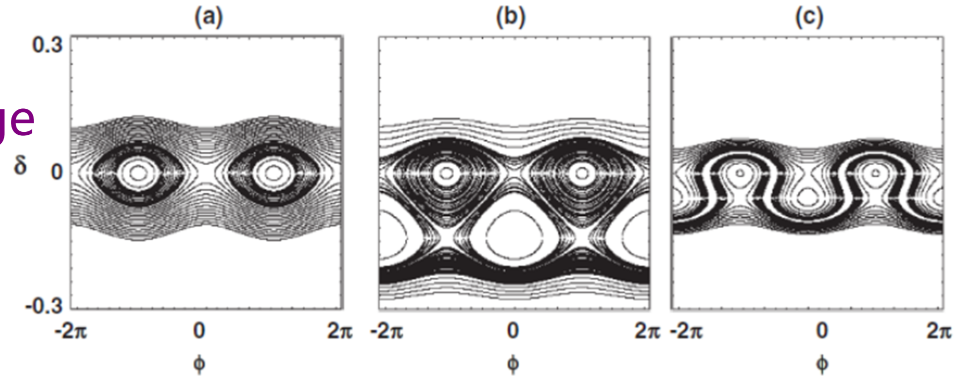


Fig. 8. Effect of second-order momentum compaction factor on longitudinal phase space in ring. The first-order momentum compaction factor in (a), (b), and (c) is 2.2×10^{-3}. The second-order momentum compaction factors in (a), (b), and (c) are 0.006, 0.02311, and 0.06, respectively.

when $|\alpha_2| > \alpha_{2Cr}$, the RF bucket will transmit to α-bucket, bucket area will shrink

$$\alpha_{2Cr} = \sqrt{\frac{E_0 h|\alpha_1|^3}{12eV_{rf}\left[-cos\varphi_s + \left(\frac{\pi}{2} - \varphi_s\right)\sin(\varphi_s)\right]}}$$

$$\eta_2(s) = -\eta_1(s) + \frac{1}{2\sin\pi Q_x} * \int_s^{s+C} \sqrt{\beta_x(s)\beta_x(\sigma)}\cos(\pi Q_x - \mu_{\sigma s}) * \left(K_1(\sigma)\eta_1(\sigma) - \frac{1}{2}K_2(\sigma)\eta_1^2(\sigma)\right)d\sigma$$

The sextupoles should be located at dispersive location to correct chromaticities for three directions

P.Gladkikh Design of laser-electron storage ring lattice dedicated to generation of intense X-rays under Compton scattering
Eun-San Kim 2007 *Jpn. J. Appl. Phys.* **46** 7952

DA optimization

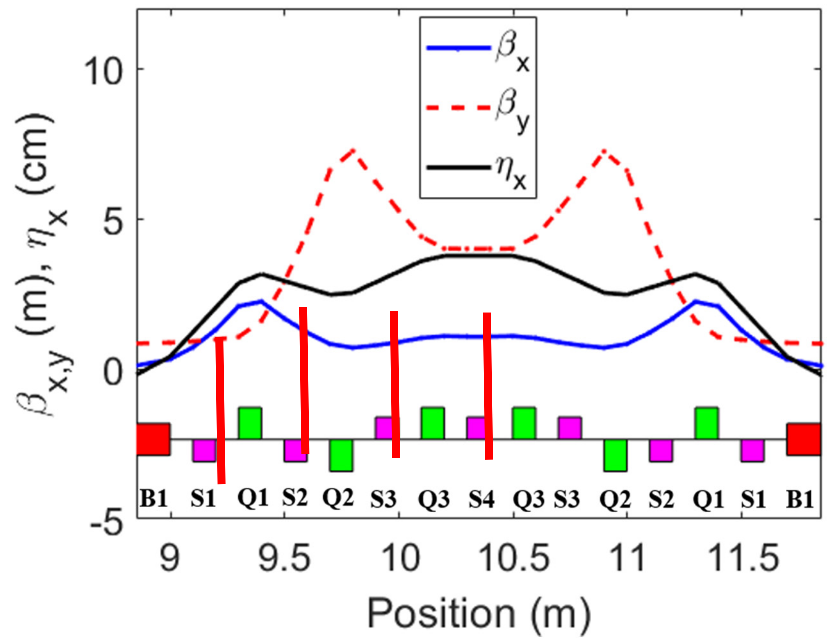
High order achromat scheme； add some geometrical sextupoles when needed

First order driving terms:

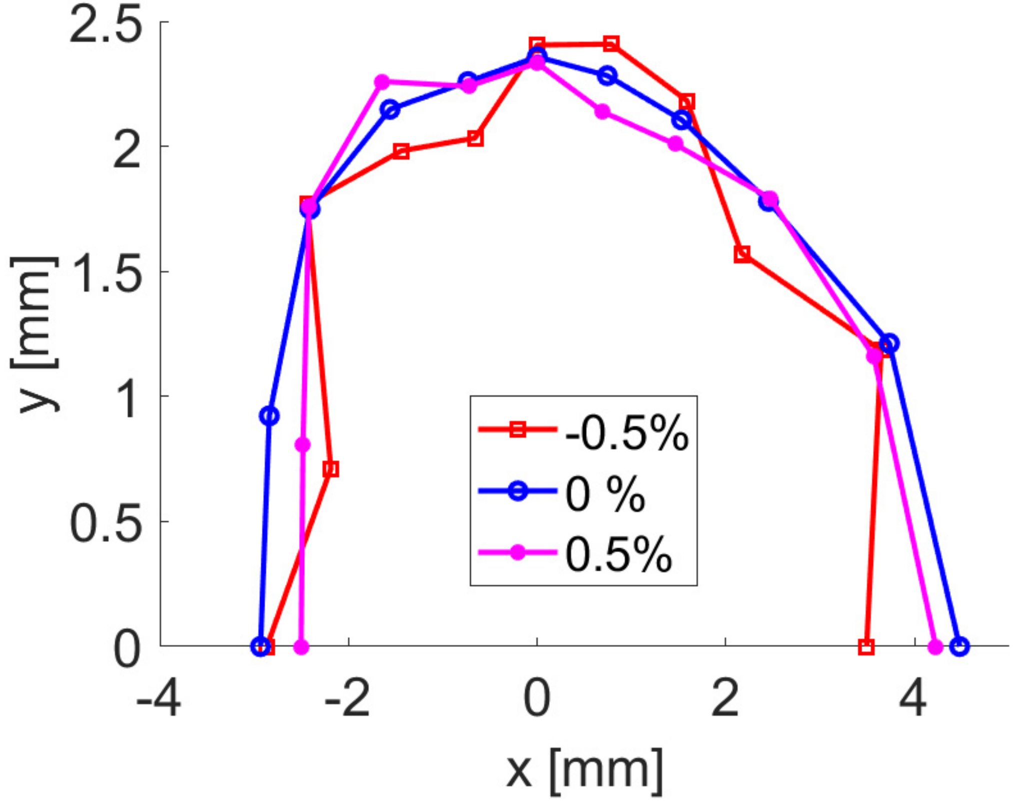
h10110	2.21e-2
h21000	6.32e-3
h30000	7.28e-3
h10020	1.08e-2
h10200	3.19e-2
h20001	6.53
h00201	5.80
h10002	3.25e-2

Second order driving terms:

h22000	7.76e4
h00220	2.87e5
h11110	1.10e5
h40000	8.16e3
h00400	1.85e5
h31000	1.97e1
h20110	1.04e2
h11200	5.23e2
h00310	2.26e3
h20020	4.35e4
h20200	1.52e4



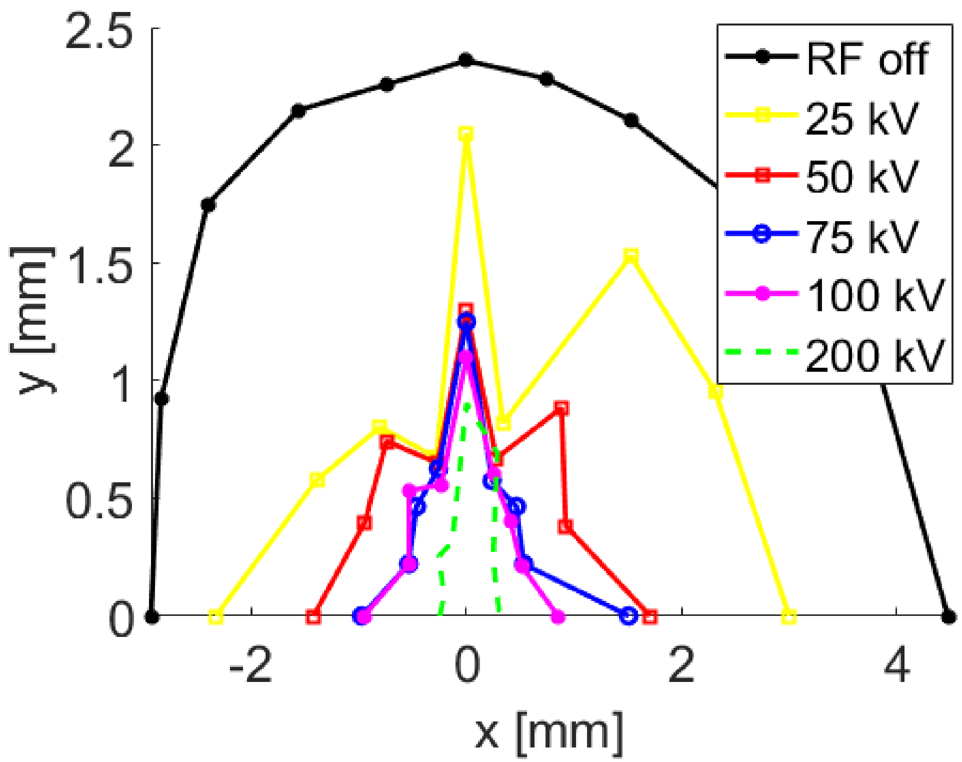
DA optimization



4-D DA， x:~3-4 mm， y:~2.5 mm

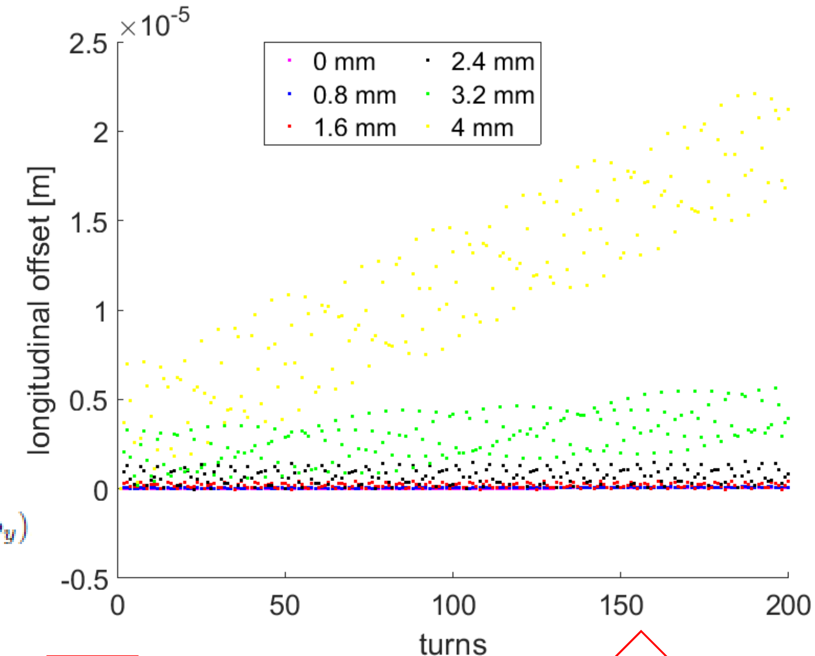
DA optimization-6D issue

RF will be replaced by laser modulator in SSMB ring, the wavelength is 1 μm, the path length oscillation amplitude by transverse longitudinal coupling can be larger than 1 μm easily. So the particles with large transverse emittance will transmit to different bucket turn by turn, which is not stable.



DA optimization-6D issue

- Typically, h20001 will be ~10 for high order achromat scheme.
- h22001 and h40001 will also be important in many cases.
- 1) Minimize some important terms by COSY & 2) using MOGA (diffusion rate)



$$H^{(n)} = \sum_{a+b+c+d+e=n} h_{abcde} J_x^{+a} J_x^{-b} J_y^{+c} J_y^{-d} \delta^e exp(i(a-b)\phi_x) exp(i(c-d)\phi_y)$$

n==3

$$dz = \frac{\partial H^{(3)}}{\partial \delta} = 2*h_{11001}*J_x + 2*h_{00111}*J_y + 2*h_{20001}*J_x*exp(i2\phi_x) + 2*h_{00201}*J_y*exp(i2\phi_y).$$

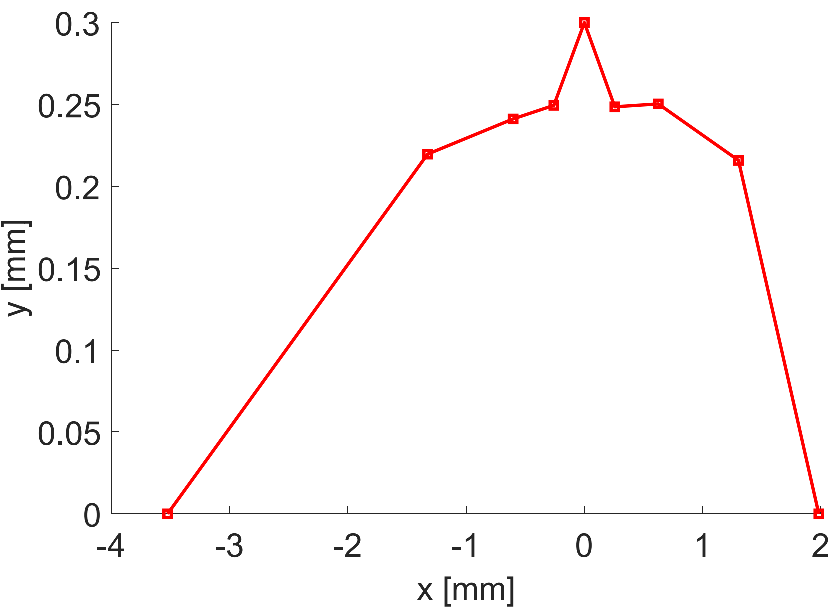
n==5

$$dz = \frac{\partial H^{(5)}}{\partial \delta} = 4*h_{22001}*J_x^2 + 4*h_{00221}*J_y^2 + 4*h_{40001}*J_x^2*exp(i4\phi_x) + 4*h_{00401}*J_y^2*exp(i4\phi_y)$$
$$+ 4h_{11111}*J_x*J_y + 4*h_{31001}J_x^2*exp(i2\phi_x) + 4*h_{13001}*J_x^2*exp(-i2\phi_x)$$
$$+ 4*h_{00311}J_y^2*exp(i2\phi_y) + 4*h_{00131}*J_y^2*exp(-i2\phi_y)$$
$$+ 4*h_{20201}*J_x*J_y*exp(-i2(\phi_y+\phi_x)) + 4*h_{30101}*J_x^{3/2}J_y^{1/2}*exp(i(3\phi_x+\phi_y))$$
$$+ 4*h_{10301}*J_y^{3/2}J_x^{1/2}*exp(i(3\phi_y+\phi_x)) + c.c..$$

RF is off; $z_0 = 0, \delta_0 = 0$

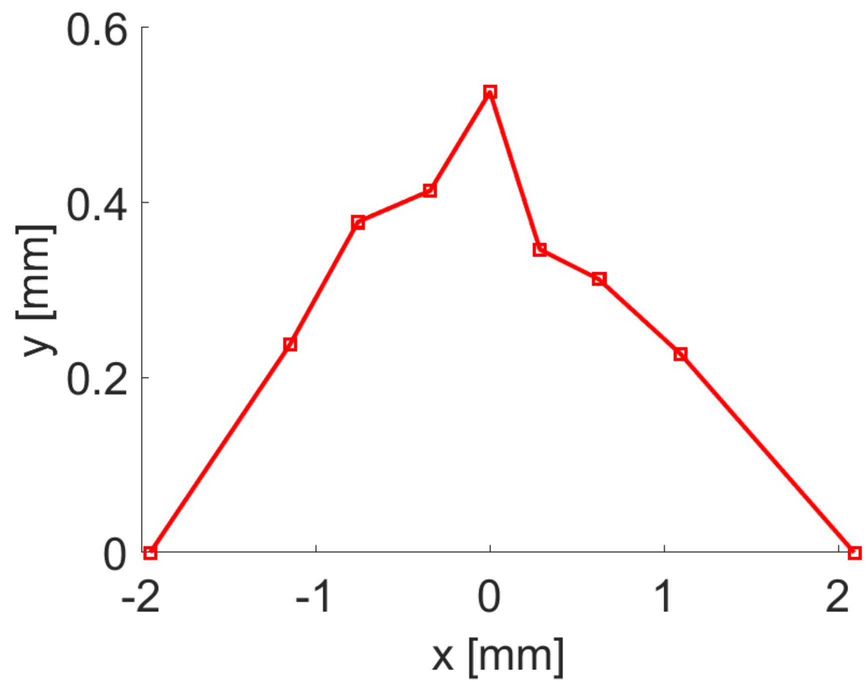
DA optimization-6D issue

COSY result



6-D DA

MOGA result



6-D DA

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- Introduction
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Summary

- We have done the analysis for partial alpha effects in low alpha ring, and proposed a method to minimize it.
- Based on the analysis, the linear lattice is designed, bunch length under 100 nm can be maintained in the ring.
- We have done some preliminary studies on the nonlinear optimization for this kind of storage ring, the 6-D DA will be limited by T-L coupling, which need more careful studies for further optimization.

Thanks!