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AUTOMATED DESIGN AND OPTIMIZATION OF THE FINAL COOLING FOR A MUON COLLIDER

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Abstract

The desired beam emittance for a Muon Collider is several orders of magnitude less than the one of the muon beams produced at the front-end target. Ionization cooling has been demonstrated as a suitable technique for the reduction of muon beam emittance. Final cooling, as one of the most critical stages of the muon collider complex, necessitates careful design and optimization in order to control the beam dynamics and ensure efficient emittance reduction. We present an optimization framework based on the ICOOL simulation code and application of different optimization algorithms, to automate the choice of optimal initial muon beam parameters and simultaneous tuning of numerous final cooling components.

INTRODUCTION

The proton-driven muon production leads to muon beams which initially occupy a large phase space. Cooling is required in order to reduce the beam emittance to satisfy the transverse and longitudinal acceptance requirements of the muon accelerator ring and to increase the luminosity of the collider, which is inversely proportional to the beam sizes [1]. An overview of the complete baseline design of the muon collider complex including different stages from proton injector to collider ring can be found in [2]. The 2.2 μs lifetime of muons makes ionization cooling [3, 4] the only possible technique to be applied to reduce the 6D emittance. In ionization cooling channels, muon beams are sent through absorber material placed at a focus. After losing energy through interaction with the absorber material, the beams pass through rf cavities which re-accelerate the particles.

In the past, extensive simulation studies have been conducted on the design of a final cooling system capable of achieving sufficiently small transverse emittance, at the cost of increasing the longitudinal emittance. The design presented in [5] aimed for transverse emittance reduction to 50 μm, while 55 μm have been achieved, increasing the longitudinal emittance from 1.5 to 70 mm at the end of the final cooling channel. However, latest studies show that transverse emittance below 25 μm is preferred [6]. Employing stronger solenoid fields and lower beam momenta will help to achieve lower emittances. This work focuses on the development of the infrastructure for design of the final stage of muon cooling, to improve the optimization process in terms of speed and usability of simulation tools.

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Simulation Tools for Ionization Cooling

Previous design studies of final cooling channel of a muon collider have been performed using either G4BEAMLINE code, which is based on GEANT4 libraries, or the simulation code ICOOL which has been developed specifically for 3-D tracking of particles in ionization cooling channels [7]. The cooling cells are defined as series of longitudinal regions with assigned configurations of different accelerating and magnet fields and material properties. The main advantage of these codes for the final cooling studies is that particle tracking takes into account decays and interactions of low energy muons in matter.

Motivation

Simple diagnostics such as calculation of emittances and optics functions are implemented in ICOOL, however the modification of text file-like input decks and evaluation of results imposes a big challenge to the users. Especially in a view of enlarging the muon collider study community, it is necessary to provide easily accessible, user-friendly software in order to reduce the effort of constructing and evaluating a large amount of simulation studies required to define an optimal design.

To achieve ideal matching between stages and various cooling channel configurations, a large parameter space needs to be explored. It is important to find an optimal trade-off between reduction of transverse emittance and longitudinal heating and transmission in order to provide beams with required properties to the next stages of muon collider complex. Apart from employing advanced optimization techniques, automation of the optimization and design is needed.

TOWARDS AUTOMATIC DESIGN

To improve the usability of the simulation framework for the final cooling, we extended it with an input handler and post-processing analysis both written in Python, which allows an easy integration of numerical optimization tools e.g. provided by the *scipy* library, but also the implementation of other advanced optimization techniques presented in the next section. The new components and their integration in the current version of optimization framework are presented in Fig. 1. The initial beam distribution and cooling channel properties are given in JSON format, which is well structured and can be easily modified manually or automatically during the optimization. The current value of a defined objective function is obtained by running ICOOL simulation

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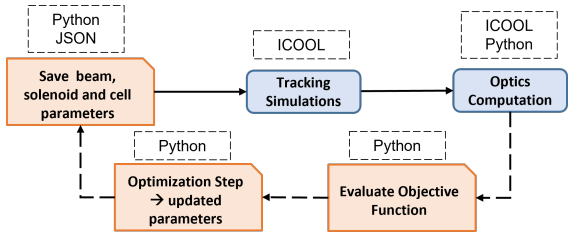

Figure 1: Optimization workflow including existing codes for particle tracking simulation and optics computation (blue) and recently developed optimization and data handling routines (orange).

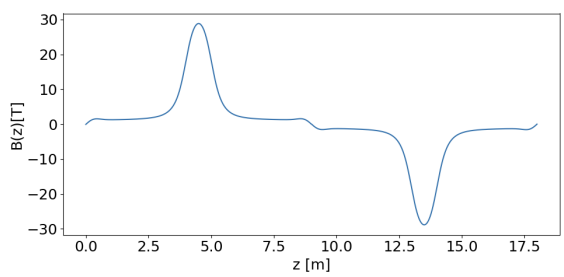

Figure 2: Strength of solenoid field inside the simplified lattice of two consecutive cooling cells with a field flip.

with the provided input parameters. A number of templates automatically translate the JSON file to an ICOOL input deck. After the tracking is performed and emittances and optics parameters are computed, the objective function is evaluated, and the simulation parameters used in the previous step are updated by a selected optimization algorithm. The new parameters are then saved and provided again to the ICOOL configuration files for the next optimization step.

The implementation in Python allows an easy exchange of optimization algorithm to compare their performances and reduces the effort of dealing with rather complex variable notations and input deck configuration in ICOOL. Another advantage of the developed extension is the possibility to systematically save the simulation results together with its configuration, such that the studies are easily reproducible and data-driven modeling and optimization can be applied in the future [8].

OPTIMIZATION VARIABLES AND TARGETS

The reduction of normalized transverse emittance depends on several properties of the cooling channel. Strong focusing at the absorber location reduces the transverse heating effect arising from multiple Coulomb scattering. Apart from the strong solenoid field at the location of absorber, special matching coils are needed at the entrance and exit of the absorber in order to reduce emittance blow-up which is otherwise caused by optics mismatch. The choice of the β-function at the absorber is defined given the strength of solenoid B and beam momentum P as $\beta_\perp(\mathrm{m}) = 2P(\mathrm{GeV/c})/0.3B(\mathrm{T})$ [5]. However, it is not possible to de-

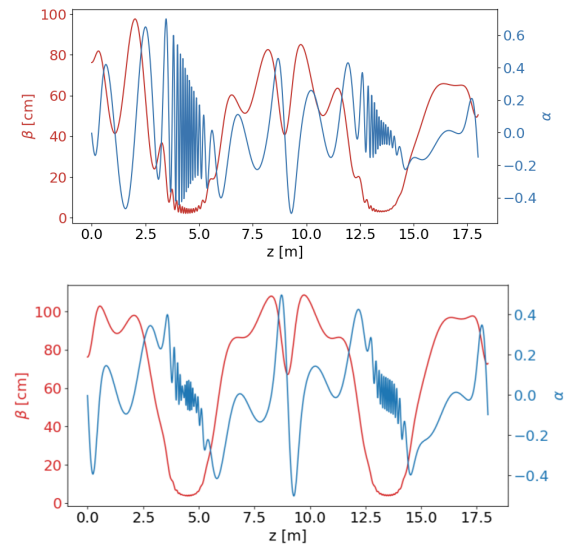

Figure 3: The optics in non-optimized solenoid field (top) leads to an emittance blow up of 15% compared to unchanged transverse emittance achieved with matched optics (bottom).

fine the initial β-function in the same way, since the field is designed to be 0 T at the beginning of the cell, as shown in Fig. 2. Hence, the β-function at the start of the lattice is one of the beam parameters to be optimized, together with its momentum. The starting values $\epsilon_\perp = 300$ µm, $\epsilon_\parallel = 1.5$ mm and $\sigma_z = 5$ cm are adopted from previous studies [5].

The lattice consists of a strong focusing solenoid, followed by matching coils to provide a focusing field starting from 0 T, with the peak strength of 30 T, with a liquid hydrogen block placed in the strong solenoid field and rf cavities for restoring the longitudinal momentum. The magnetic field is defined as a set of cylindrical current sheets, providing current density, length and radius of each coil. Based on this configuration, further optimization variables are used: the radii of high field solenoid and matching coils, absorber length, rf length, frequency and gradient. For two cooling cells, combined with a field flip to limit the accumulation of the canonical angular momentum [9], 19 free parameters must be optimized. Depending on the optimization target, a reduced variable set can be used. In the following, we consider two possible strategies for the optimization - simplified case of optics matching in the absence of absorber and re-acceleration, and an integrated optimization towards transverse emittance reduction in two consecutive cooling cells, including absorbers and rf-cavities.

Applied Optimization Techniques

In order to identify the most suitable optimization strategy, we applied different numerical optimization approaches to the simplified problem of optics matching. One of the standard optimization algorithms, Nelder-Mead [10], is known to be robust in many applications, however it becomes inefficient in high dimensions and does not allow parallelization

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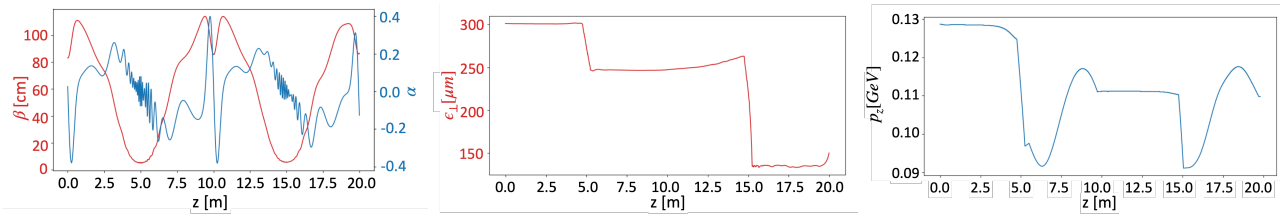

Figure 4: Optimization of transverse emittance reduction in a cell with a hydrogen block and rf for phase-space rotation.

in its existing implementations. Differential evolution algorithm [11], a stochastic population-based method, demonstrates better results with the same number of optimization steps compared to Nelder-Mead, however requires more time despite parallelization due to the permutation and crossover operations. The Extremum Seeking (ES) algorithm, an adaptive method developed for the control of time-varying systems [12], is the most efficient. An important advantage of ES is the capability to deal with large parameter spaces at relatively low computational and time cost, which will become more critical with growing complexity of final cooling design studies. The next section presents results using ES algorithm.

LINEAR OPTICS OPTIMIZATION

Nearly ideal matching is required for an efficient transverse emittance reduction, which can be achieved by tuning the radii of the coils forming the solenoidal field in the cooling cell. At every optimisation step, ICOOL simulation is executed with the initial beam distribution according to $\epsilon_\perp = 300$ μm, $p_z = 135$ MeV/c, and $\beta_\perp = 0.78$ m, computed using the given beam momentum and strength of the main solenoid. Fig. 3 shows the improvement on optics matching compared to initially selected coils parameters, using ES for only 20 optimization steps. The objective function to be minimized is defined as $\bar{\alpha} + |\beta_{\text{ideal}} - \beta_{\text{sim}}|$, where the absolute deviation of β_{sim} from β_{ideal} is given in [m]. The field settings found by ES algorithm allow to keep constant β-function matched to the solenoid field in the region where the absorber will be placed. Improvements are possible by tuning also the initial β-function and momentum values, as will be shown below. To be noted that including the absorber into simulations will require a re-adjustment of matching coils in each cell individually, due to the change of beam momenta after exiting the absorber region.

TRANSVERSE COOLING

Next, we consider a more complex simulation set up, including a drift space following the absorber and rf cavity set to have zero phase for longitudinal phase-space rotation. The target is to simultaneously optimize two cooling cells towards transverse emittance reduction, while keeping the transmission high and restoring the longitudinal momentum, maintaining the longitudinal emittance below 70 mm. The optimization of accelerating rf cavities frequencies and phases to match the bunch length will be performed as the next step of the current studies. In the case when a trade-off between several objectives is required, the definition of the objective functions becomes more challenging. Here, we are using the following function to be minimized by varying parameters p: $\min_p[\frac{\Delta\epsilon_\parallel}{(\Delta\epsilon_\perp \Delta N)} + \bar{\alpha}]$, where ΔN is the relative transmission rate and $\Delta\epsilon_\parallel$, $\Delta\epsilon_\perp$ the relative change in longitudinal and transverse emittance, respectively.

As mentioned above, the full set of optimization variables consists of 19 parameters. The results achieved with ES algorithm performing 75 steps are summarized in Fig. 4. Considering the optics in the absence of cooling and re-acceleration, slightly better matching is achieved compared to the simplified case presented above. Adding hydrogen absorber with the lengths 60 cm and 40 cm in first and second optimized cell respectively results in the transverse emittance reduction of 50%. The different absorber lengths defined by optimization algorithm can be explained with the beam energy change after the first cell, such that shorter absorber is preferred in the second cell for the trade-off between emittance reduction and transmission. The longitudinal momentum is then restored up to 85 % of its initial value, with transmission of 78 % and longitudinal emittance of 20 mm.

CONCLUSION

We investigated an automatic optimization scheme for the design of final cooling for the muon collider, based on the recently developed extension to ICOOL simulation code and ES tuning algorithm. First application demonstrates promising results in terms of optics control and transverse emittance reduction in a simplified cooling lattice consisting of two cells. Further improvements are possible by individual matching of each cell, adjusting to the momentum change of cooled beams. A potential strategy to push the cooling performance beyond the previously achieved values is to propagate the parameters starting from the last cooling cell before linac acceleration, in order to ensure that desired beam parameters are achievable, considering stronger solenoid fields and alternative absorber configurations.

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