

# RECONSTRUCTION OF TRANSVERSE PHASE SPACE FROM TRANSVERSE FEEDBACK DATA FOR REAL TIME EXTRACTION OF VITAL LHC MACHINE PARAMETERS

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## Abstract

The LHC transverse feedback system (ADT) provides bunch by bunch, turn by turn, normalized and digitized beam position signals from four pick-ups per plane and for each beam. Together with already existing powerful computer-based observation systems, this data can be used to reconstruct in real-time the transverse phase space coordinates of the centre-of-charges, for each individual bunch. Such information is extremely valuable for machine operation, or transverse instability diagnostics.

This paper aims on discussing and evaluating methods of combining four position signals for such analysis in the presence of noise and with active transverse feedback. Comparisons are made based on the extraction of vital parameters like the fractional tune or transverse activity. Analytical and numerical results are further benchmarked against real beam data.

## TRANSVERSE FEEDBACK AND ADTOBSBOX

During the LHC Long Shutdown II the LHC transverse feedback system (ADT) [1] was subject to an upgrade of its Beam Position Monitor (BPM) hardware, aiming for an improvement of the system's noise floor [2]. New low-noise beam position hardware is now available for LHC Run III, providing independent processing of 16 dedicated pick-ups. This allows for a total of four beam position streams per plane and per beam, representing the transverse centre-of-charges of each individual bunch.

This data is available in real-time to the LHC Transverse Feedback system for damping and stabilizing the beam, as well as to the computer-based observation system ADTObsBox [3]. The ADTObsBox is capable of recording and processing all digital data streams from the available pick-ups at full data rate (bunch-by-bunch or 40 Msps, with 16 bit resolution), for instance, to combine the data in real-time and extracting valuable metrics of the ADT performance (e.g. transverse activity) or the beam itself (bunch-by-bunch fractional tune).

One considerably useful representation of transverse motion is the use of phase space coordinates, readily described in Ref. [4] as an analytic signal  $x[n]$ , expressed as,

$$x[n] = y[n] + jy'[n]. \quad (1)$$

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Here, the transverse normalized phase space coordinates  $y[n]$  and  $y'[n]$  represent normalized position data respectively the corresponding normalized slope values at turn index  $n$ .

From the notation in Eq. (1) we can immediately derive a measure of transverse beam activity, denoted by  $A[n]$ , as the modulus – or amplitude – of the complex vector,

$$A[n] = \text{abs} \{x[n]\}. \quad (2)$$

If  $A[n]$  decreases over time then the transverse activity is considered to be damped, whereas growing values provide an indication of transverse instability.

Equivalently, the fractional tune,  $Q[n]$ , follows from phase space data by observing the phase component  $\varphi[n] = \arg \{x[n]\}$  of Eq. (1), specifically, the relative phase advance between consecutive turns, described as,

$$Q[n] = \frac{1}{2\pi} \arg \left\{ \frac{x[n-1]}{x[n]} \right\} = \frac{1}{2\pi} (\varphi[n-1] - \varphi[n]). \quad (3)$$

For the analytic evaluation, we implicitly assume that the beam is centred in the pick-ups and we observe betatron oscillations, i.e.  $(y)^2 + (y')^2 = \text{const}$ . Furthermore, for our assessment, we shall use a damped, complex-valued harmonic oscillator as beam model, which provides for the  $k^{\text{th}}$  beam position monitor at the  $n^{\text{th}}$  turn the phase space coordinates as follows,

$$x_k[n] = A_0 e^{-j\phi_k} (\alpha \cdot e^{-j\omega_0})^n. \quad (4)$$

Here,  $A_0$  and  $\phi_k$  are initial conditions,  $\alpha$  accounts for an amplitude decay and  $\omega_0 = 2\pi Q_f$  represents the angular frequency at the fractional betatron tune  $Q_f$ .

## PHASE SPACE RECONSTRUCTION

In order to satisfy Eq. (1), and by acknowledging that the ADT Beam Position Monitors readily provide normalized readings, we are looking for indirect methods to obtain slope samples from beam position measurements.

In the following, we evaluate two methods for combining beam position data of four LHC pick-ups, identified as a *spatial* and a *temporal phase shift* in Ref. [5]. The first combines the information of several beam position monitors based on their longitudinal distribution in the accelerator, the later relies on processing the history of recorded beam position data using filter kernels.

### Spatial Method

This method relates the data measured by two or more independent beam position monitors at different longitudinal azimuths. Fig. 1 outlines the case for  $N = 2$  signal sources. For this method, the beam position monitors should have a betatron phase advance ideally between  $60^\circ < (\phi_2 - \phi_1) < 120^\circ$ .

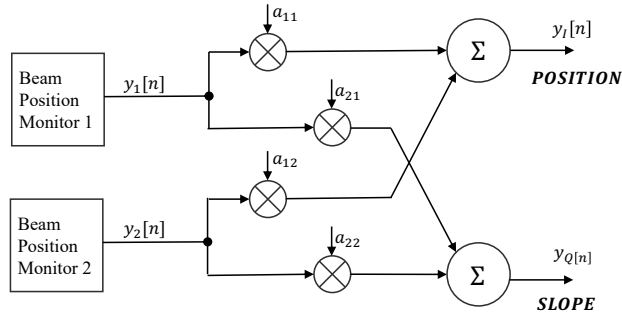


Figure 1: Spatial phase space reconstruction: in-phase term  $y_I[n]$  and quadrature component  $y_Q[n]$  calculated from vectorial rotation and combination of two beam position sequences  $x_1[n]$  and  $x_2[n]$ .

For the case of the LHC ADT, the data provided by individual beam position monitors  $y_k[n]$  are meticulously time-aligned during setting-up, with index  $n$  corresponding to the same bunch data at the same turn. Therefore, using the ‘‘Pick-up Vector Sum’’ algorithm from Ref. [5] it can be shown that, choosing appropriate mixing coefficients  $a_{11}$  and  $a_{12}$ , the position signal  $y_I[n]$  in Fig. 1 of a virtual beam position monitor can be constructed, such that

$$y_I[n] = a_{11}y_1[n] + a_{12}y_2[n]. \quad (5)$$

Equivalently, applying the same approach of pick-up signal mixing, and by taking into account an additional  $90^\circ$  phase advance compared to the virtual beam position used for Eq. (5), we obtain the representation of the slope,  $y_Q[n]$ , described as

$$y_Q[n] = a_{21}y_1[n] + a_{22}y_2[n]. \quad (6)$$

The pair of Eq. (5) and (6) represent Cartesian coordinates, describing the reconstructed phase space for an arbitrary longitudinal position. It is a convenient practice to chose the ADT kicker position as reference for the phase space reconstruction. By using the same longitudinal reference this technique is further expandable for using multiple beam position monitors – four in the case of the LHC ADT – as shown in Fig. 2.

We therefore obtain an analytic signal  $x_S[n] = y_I[n] + jy_Q[n]$  as the reconstructed normalized transverse phase space based on a scalar combination of real-valued position sequences,  $y_k[n]$ , using

$$x_S[n] = h_0[n] * \sum_k (a_{1k} + ja_{2k}) \cdot y_k[n]. \quad (7)$$

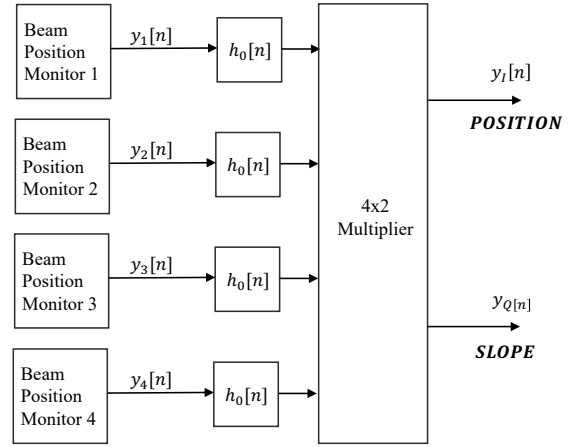


Figure 2: Spatial combination of four beam position streams.

The asterisk operator ( $*$ ) represents the discrete-time convolution of the weighted BPM signals with an additional finite impulse response (FIR) filter of impulse response  $h_0[n]$ , allowing shaping of the input noise. When no filter is used, this method has zero group delay and therefore it is suitable for applications requiring low latency.

### Temporal Method

Referring to [4], this reconstruction method obtains the slope signal as the  $90^\circ$  phase rotated version of the position data utilizing digital filters.

As outlined in Fig. 3, bunch-by-bunch data  $y_k[n]$  provided from individual Beam Position Monitors passes through a pair of matched filter kernels, denoted as  $h_I[n]$  as  $h_Q[n]$ . The filters’ even and odd symmetric impulse responses generate in-phase ( $I$ ) and quadrature output signals ( $Q$ ), which can be combined afterwards thanks to an identical group delay.

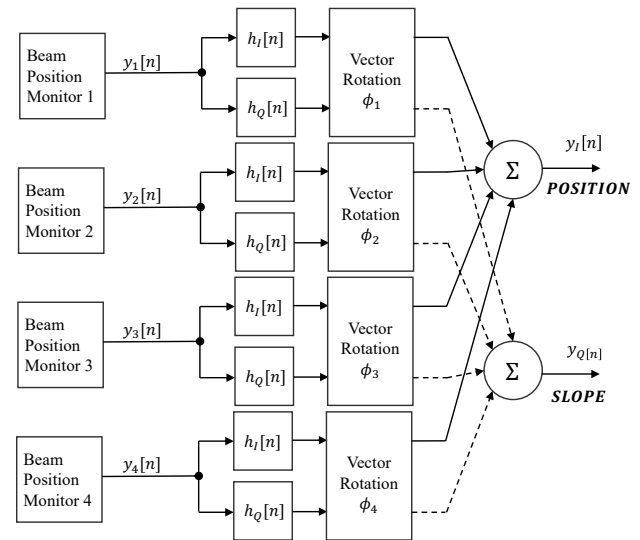


Figure 3: Temporal phase space reconstruction using matched digital filters  $h_I[n]$  and  $h_Q[n]$  on the input data stream  $y_k[n]$  of four Beam Position Monitors.

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It is worth noting that the calculated FIR filter output pairs readily represent phase space coordinates at the longitudinal position of the corresponding monitor. Therefore, to aggregate four pick-ups to an arbitrary longitudinal reference location, the individual output vectors need to be aligned before summing their contributions. This is done by a vector rotation,  $\phi_k$ , towards a common longitudinal position (e.g. to the location of the ADT kicker).

The analytic signal  $x_T[n] = y_I[n] + jy_Q[n]$  obtained from the temporal method can therefore be described as,

$$x_T[n] = \sum_k (y_k[n] * (h_I[n] + jh_Q[n]) e^{j\phi_k}). \quad (8)$$

With a short 5 taps kernel length, these filters are tuned to exhibit nominal transmission at the fractional tune for the corresponding plane, rendering them applicable for a tune range exceeding  $\pm 0.02$  around the target tune [4].

## RESULTS

To evaluate the expected performance we model the inherent noise from each Beam Position Monitor as an independent additive white Gaussian noise process,  $e[n]$  ( $e[n] = 0$ , noise power  $e^2[n] = \sigma_e^2$ ), which is added to the desired noise-free position signal,  $y_p[n]$ ,

$$y[n] = y_p[n] + e[n]. \quad (9)$$

The convolution in Eq. (7) and Eq. (8) with selective FIR filters lets signals close to the target tune pass unaltered in amplitude, but shapes the noise power for out-of-band signals. For the example of the in-phase filter (equivalent for the quadrature component), the noise power follows from,

$$\sigma_I^2 = \sigma_e^2 \sum_m |h_I[m]|^2. \quad (10)$$

The overall noise contribution is further reduced by the superposition of  $N = 4$  Beam Position Monitors, leading to a final RMS noise amplitude,

$$\sigma_A = \sigma_e \sqrt{\frac{\sigma_I \sigma_Q}{N}}. \quad (11)$$

It can be shown that the activity defined in Eq. (2) with noise present according to Eq. (9) follows the well known Rice probability distribution. The expected RMS amplitude noise follows the RMS input noise, reduced by filtering and by the combination of 4 BPMs. For example, in the case the filter kernels listed in [4] are used for the temporal method, then the expected total process gain is 9.2 dB.

The measurement noise also affects the observable fractional tune (Eq. (3)) from the reconstructed phase space data  $x_T[n]$ , with the RMS phase noise given as,

$$\varphi_{\text{rms}} = \text{atan} \frac{\sigma_A}{A_0} \quad (12)$$

At this point it is worth noting that, for large amplitudes  $\overline{A[n]} \gg 10\sigma_A$ , the reconstructed values for amplitude and

phase exhibit measurement noise following the assumed normal distribution (standard deviations  $\sigma_A$  respectively  $\varphi_{\text{rms}}$ ), however, due to the filtering their spectral density is no longer white.

Furthermore, for  $\overline{A[n]} \ll 10\sigma_A$  the observed phase vector resembles a uniform distribution, with an arbitrary phase value between  $[-\pi, \pi]$ , providing no useful information on the fractional tune. Numerical simulations using Eq. (4) and a BPM noise floor of  $\sigma_e = 0.219$  m (taken from [2]) indicate that a minimum signal to noise ratio (SNR) of  $20 \log_{10}(A[n]/\sigma_e) \geq 30$  dB is required to obtain an RMS tune variation of  $\sigma_Q$  better than  $10^{-3}$ .

This value is confirmed by measurements of LHC injection transients, shown in Fig. 4 as an example, where the turn-by-turn fractional tune of one individual bunch is reconstructed from 4 LHC BPMs using the method described by Eq. (8). The red trace indicates the 16-turns moving average of the instantaneous fractional tune  $Q_T[n]$  (in gray), and with the RMS tune error depicted in black. Between 2000 and 2500 turns after injection the oscillation amplitude has decayed to an SNR of approx. 30 dB, and the measured RMS tune variation  $\sigma_Q = 1.1 \cdot 10^{-3}$ , which is in excellent agreement with the numerical prediction.

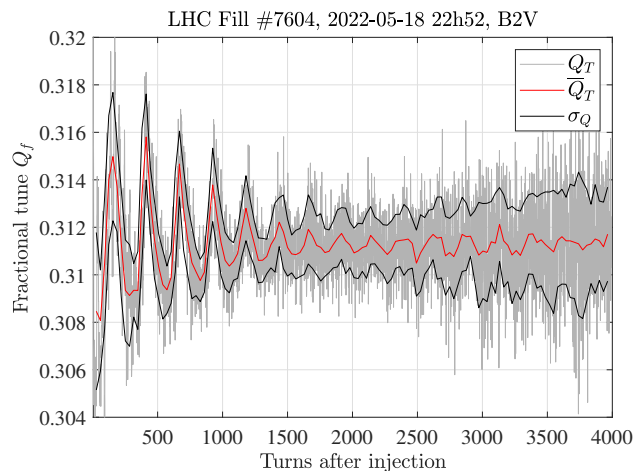


Figure 4: Fractional tune from reconstructed transverse phase space using 4 LHC BPMs using the temporal method.

## CONCLUSION

The suggested methods are both valid candidates for reconstructing the transverse phase space in real-time. With the spatial method being attractive for the analysis of fast beam transients, for example during injection transients and with 5 turns damping time, both methods profit from the suppression of out-of-band noise using filter kernels which improves the SNR.

Potentially, more elaborate filter response functions can be designed to lower the SNR even further, for instance by processing the beam position data of 100 to 1000 turns.

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