# TRACKING DYNAMIC APERTURE IN THE iRCMS HADRONTHERAPY SYNCHROTRON* 

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## Abstract

Dynamic aperture (DA) studies which are part of the ion Rapid Cycling Medical Synchrotron (iRCMS) lattice design have been undertaken. They are aimed at supporting ongoing plans to launch the production of the six magnetic sectors which comprise the iRCMS racetrack arcs. The main bend magnetic gap is tight (see Appendix), so allowing smaller volume magnets and resulting in a compact ring. The DA happens to be commensurate with the mechanical aperture, thus tracking accuracy is in order. In that aim, DA tracking uses the OPERA field maps of the six 60 degree magnetic sectors of the arcs. Simulation outcomes are summarized here.

## INTRODUCTION

The $400 \mathrm{MeV} / \mathrm{u}$ Carbon iRCMS is a 2-periodic racetrack layout, its $180^{\circ}$ arcs are achromats, comprised each of three $60^{\circ}$ sectors separated by 1.14 m long drifts with include a quadrupole at their center (Fig. 1). Each sector is a BDH-BF-BD-BF-BDH combined function (CF) dipole series (Fig. 2, details in the Appendix), the dipole parameters are given in Table 1. One of the LSS accommodates two RF stations, sized for 15 Hz rapid-cycling. The other LSS accommodates the $8 \mathrm{MeV} / \mathrm{u}$ beam injection and the extraction systems. Five quadrupoles are installed in each of the two 13.75 m long straight sections (LSS) of the racetrack, they serve to set the tunes, nominally in the region $v_{x}=4.8, v_{y}=4.4$. Table 2 summarizes the optical parameters of the ring, Fig. 3 displays the optical functions.

One $60^{\circ}$ sector prototype has been manufactured, at present under AC magnetic field measurements at the magnet division of Brookhaven National laboratory (BNL) ${ }^{1}$.


Figure 1: iRCMS ion ring.

* Work supported by a TSA agreement between Best Medical International and Brookhaven Science Associates, LLC under Contract No. DE-AC0298CH10886 with the U.S. Department of Energy. fmeot@bnl.gov
${ }^{1}$ Under a Technical Scientific Agreement (TSA) with Best Medical International (BMI).

A good agreement was found between the DC magnetic field measurements of the prototype $60^{\circ}$ sector and the magnetic fields obtained from its OPERA model [1], confirming in particular magnetic lengths and focusing strength (Table 1). However, ray-tracing using the OPERA field maps showed that the 0.9 deg spacing (about 8 cm along the $\mathrm{R}=508 \mathrm{~cm}$ arc, see Appendix) between the magnets causes a 10 mm scalloping of the closed orbit around the $\mathrm{R}=508.022 \mathrm{~cm}$ reference along the sector [2]. This is not acceptable as it would eat up $20 \%$ of the $\pm 2.5 \mathrm{~cm}$ horizontal aperture (Table 1) and 10 mm of a tight DA. As a consequence the sector has been re-designed, F and D-type dipole lengths have been re-optimized so that the former 0.9 deg spaces are filled with additional F and D -style laminations, leaving a marginal 0.034 deg spacing ( 3 mm along the $\mathrm{R}=508 \mathrm{~cm}$ arc), Fig. 2 [3]. In passing, this allowed salvaging the girder, main coil, and individual dipole tuning coils of the original design.


Figure 2: New design of the iRCMS 60 deg sector, with essentially null spacing between the CF dipoles, in order to ensure constant dipole field along the $\mathrm{R}=508.022 \mathrm{~cm}$ design arc and thus a circular reference orbit and maximal horizontal geometrical acceptance.

Table 1: Parameters of the CF Dipoles. Magnet current is of the form $I=I_{\text {inj }}+I_{0} \sin (215 t)$ for a 15 Hz cycling

| Magnet | Arc-Length [deg] | Bend Angle [deg] | $\begin{aligned} & \hline \text { K1 } \\ & \left\|\mathbf{m}^{-2}\right\| \\ & \hline \end{aligned}$ | Comments | Hor. Apert. $[\mathrm{cm}]$ | Vert. Apert. [cm\| |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BDH | 7.2 | 7.2 | 1.477 | defocusing | $\pm 2.5$ | $\pm 1.75$ |
| BF | 14 | 14 | 1.477 | focusing | $\pm 2.5$ | $\pm 1.75$ |
| BD | 14 | 14 | 1.477 | defocusing | $\pm 2.5$ | $\pm 1.75$ |
| BF | 14 | 14 | 1.477 | focusing | $\pm 2.5$ | $\pm 1.75$ |
| BDH | 7.2 | 7.2 | 1.477 | defocusing | $\pm 2.5$ | $\pm 1.75$ |

Regarding the RF system, two ferrite loaded cavities have been built; measured gap voltages is 15 kV over the full RF range of $0.6-3.35 \mathrm{MHz}$ while sweeping at 15 Hz . These two single-gap RF stations therefore provide 30 kV per turn, more than necessary; synchronous phase during acceleration is in 30 deg region. These are the typical parameter values liable to be considered for DA estimates.

Table 2: iRCMS Parameters

| Circumference [m] | 64 |
| :--- | :---: |
| Number of cells in the arcs | 6 |
| Combined function magnets per cell | 5 |
| Quadrupole magnets per Straight Section | 5 |
| Horizontal/Vertical tunes | $4.84 / 4.41$ |
| Max Hor./Vert. beta functions [m] | $12.16 / 9.44$ |
| Max Hor. dispersion function [m] | 1.55 |
| Hor./Vert. chromaticity $\xi_{x, y}$ | $-5.3 /-5.12$ |
| Transition $\gamma_{t}$ | 4.2 |
| Repetition rate $[\mathrm{Hz}]$ | 15 |



Figure 3: Optical functions of the iRCMS ring.


Figure 4: Left: dipole field along $\mathrm{R}=508 \mathrm{~cm}$, and quadrupole field along $\mathrm{R}=509 \mathrm{~cm}$, in a 60 deg sector. Right: a fragment of $B_{y}$ OPERA field map in the 0.034 deg transition region between a F and a D dipole.

## DYNAMIC APERTURE

High resolution field maps of the 60 deg sectors are used to track ring DA and allow accurate DA assessment from tens of thousands of turns tracking. A sample of the field across the 0.034 deg transition region between a F and a D dipole is displayed in Fig. 4. Table 3 gives three sets of different $\mathrm{BH}, \mathrm{BD}$ and BF dipole lengths, which have been computed in OPERA (namely, three different field maps). Dynamic aperture properties are investigated for each of these three field maps - however, given the small difference in lengths between the three sets, essentially no difference is expected regarding DAs.
The elementary step of DA computation in the present approach, consists in finding the maximum vertical stable phase space invariant, for a given horizontal invariant value. The exercises is repeated for a set of horizontal invariant

Table 3: Three sets of lengths of the 60 deg sector magnets that generate achromatic 180 deg arcs

|  | Arc Length [deg] |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Case \# | BDH | BF | BD | BF | BDH |
| $\mathbf{1}$ | 7.09445 | 15.000 | 15.8111 | 15.000 | 7.09445 |
| $\mathbf{2}$ | 7.5 | 15.24565 | 14.5087 | 15.24565 | 7.5 |
| $\mathbf{3}$ | 7.35323 | 15.14677 | 15.000 | 15.14677 | 7.35323 |

values taken over the horizontal stability range. Typical such maximum horizontal and vertical stable invariant search trials are displayed in Fig. 5.


Figure 5: Search for maximum stable amplitude. Left: 10,000-turn, horizontal with subliminal initial vertical motion, $v_{x}=5.30, v_{y}=4.43$. Right: vertical for some particular initial horizontal invariant value, $v_{x}=5.73, v_{y}=4.28$. Observation location is at the center of the LSS, where H and V phase space ellipses are upright.


Figure 6: Left: large dynamic aperture at 10,000 turns, in the region $\mathrm{Qx} / \mathrm{Qy}=5.15 / 4.19$. Right: 1,000-turn DA in $\mathrm{Qx} / \mathrm{Qy}=4.15 / 4.28$ region.

The set of points so obtained, namely, maximum stable invariant in vertical phase space as a function of horizontal invariant value from paraxial to maximum horizontal stable amplitude, represents the DA. The maximum stable amplitude is assessed here over a few thousands of turns, up to 10,000 . This is considered sufficient as the bunch is captured during the 15 Hz acceleration ramp, so that its size quickly damps (the acceleration from $8 \mathrm{MeV} / \mathrm{u}$ injection energy to top energy takes a few $10^{4}$ turns).

Sample DA results are given in Fig. 6, at constant energy. DA samples in Fig. 7 account for acceleration. It is not a bad idea to keep an idea on the impact of turn number (Fig. 8).

Following these preliminary estimates, tune scans are in preparation for a systematic study.

However the 10,000 samples displayed here indicate great enough DA with properly chosen working points.


Figure 7: Effect of synchrotron motion, 3,000-turn DA. Top left: accelerated with synch motion; top right: accelerated without; $\mathrm{Qx} / \mathrm{Qy}=5.15 / 4.2$ region. Bottom: accelerated with synchrotron motion, $\mathrm{Qx} / \mathrm{Qy}=5.2 / 4.15$ region.


Figure 8: Dependence of DA on number of tracking turns.

## APPENDIX

- Details regarding the iRCMS prototype 60 deg sector under AC measurements at BNL [1]


Prototype 60 deg sector. The tuning coils are apparent.

(A fragment of the) prototype 60 deg sector geometry, with its 8 cm magnet spacing (left), and the resulting field dip along that spacing (right).

- Details regarding the iRCMS combined function dipoles [3]


Focusing (left) and defocusing magnet (right) in OPERA.


Vacuum pipe in the CF magnet tight gap.

## REFERENCES

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