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A FIRST ATTEMPT AT IMPLEMENTING TRIBs IN BESSY III'S DESIGN LATTICE

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Abstract

At HZB's BESSY II and PTB's Metrology Light Source (MLS), resonances and islands in transverse phase space are exploited in a special operation mode usually referred to as Transverse Resonance Island Buckets (TRIBs). This mode provides a second stable orbit well separated from the main orbit and one of its applications in photon science is the ultra-fast switching of the helicity of circularly polarized light pulses. In the context of the conceptual design study of BESSY III, it is under investigation how this special optics mode can be implemented in an MBA structure and how it will impact the photon source parameters. In this paper we present a preliminary attempt at implementing TRIBs in BESSY III's design lattice, a multi-bend achromat, by breaking the symmetry of the lattice.

INTRODUCTION

The design work of BESSY III as BESSY II's successor started in 2020 [1] and it will be a 4th generation lightsource aiming for highest brilliance and an equilibrium emittance of 100 pm rad. It is well established that the state of the art solution for this requirement is a low-emittance multi-bend achromat (MBA) lattice. Two preliminary MBA lattice candidates have been proposed for BESSY III which we will name SF and CF and their respective lattice parameters are summarized in table 1.

Table 1: Global Parameters of the SF and CF Lattices

	SF	CF
Energy	2.5 GeV	2.5 GeV
Circumference	346 m	344 m
Natural emittance ε_0	99 pmrad	98 pmrad
Straight sections	16 × 5.6 m	16 × 5.6 m
Tune (Q_x, Q_y)	(44, 13)	(44, 13)
Mom. comp. factor α	10^{-4}	10^{-4}
Chromaticity (ξ_x, ξ_y)	(-94.7, -37.3)	(-83.1, -57.4)

The SF lattice contains homogeneous dipoles in the achromat and combined function dipoles in the matching cell and vice versa for the CF lattice. Both lattices were designed using the higher-order achromat (HOA) scheme [2, 3] which is robust against engineering tolerances such as misalignments and focusing errors. More details on the lattice development of BESSY III can be found in [4, 5]. The CF lattice is standard practice for MBA projects but in what follows, we will adopt the SF lattice as the baseline lattice [4] and its optics is shown in Fig. 1. The horizontal and vertical beta functions are β_x and β_y respectively and η_x is the horizontal dispersion. The only nonlinear elements present in the lattice are the chromatic sextupoles adjusted to correct the chromaticity to $(\xi_x, \xi_y) = (0, 0)$. All simulations were carried out in Elegant [6].

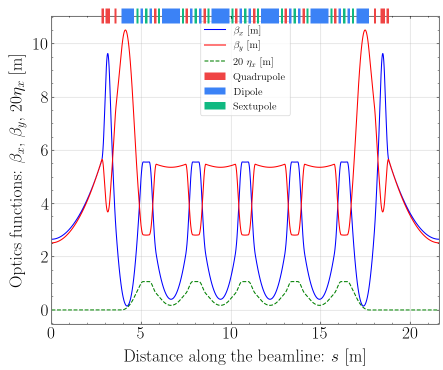

Figure 1: Superperiod of the SF lattice.

Timing mode experiments [7,8] are essential at BESSY II and the MLS and with TRIBs, a second stable orbit is operated together with the main one which can have a different fill pattern [9, 10]. This is beneficial to users because they can be provided light pulses with different repetition rates at the same beamline and a practical application at BESSY II in photon science is the fast switching of light polarization [11]. Due to its successful operation and benefits at BESSY II, the possibility of the TRIBs mode for BESSY III is appealing and worth investigating.

SYMMETRY BREAKING AND RESONANCES

Usually, storage rings are designed with repetitive/periodic structures, namely superperiods but engineering tolerances such as misalignments and focusing errors break the symmetry of the lattice and excite resonances. This also means that islands are always present in more realistic lattices (at least in principle) because they are resonance phenomena but since they are unwanted and considered to lie outside the dynamic aperture (stability region), working points are usually chosen to avoid them. As a first check, the SF ring was tuned to $(Q_x, Q_y) = (44.38, 13.2)$ using two quadrupoles in the triplet section to check whether islands in the horizontal (x, p_x) phase space can be established when the horizontal tune Q_x hits the third-order $(1/3)$ resonance.

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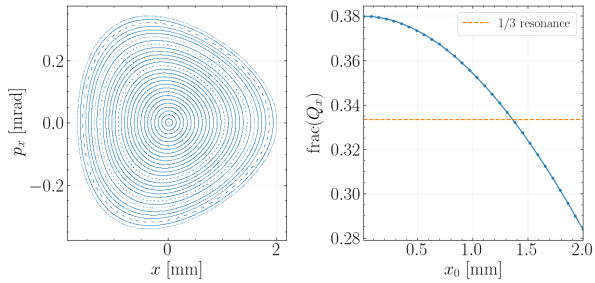

Figure 2: Horizontal phase space and FFT for periodic lattice.

Figure 2 shows the tracking result (2048 turns) for different initial offsets $\{x_0\}$ as well as the fast Fourier transform which gives the (fractional) tune-shift with amplitude (TSWA). We can see that, although the $1/3$ resonance is crossed, it is not excited and islands are absent.

It has been observed previously in numerical experiments that third-order islands cannot be established in a few design lattices with sextupoles i.e. a tune-shift with amplitude (TSWA) is a necessary but not sufficient condition; lattice perturbations are also required [10]. This nontrivial fact is usually accepted as an empirical rule, but here, we sketch a possible general explanation based on familiar analytical methods, namely the formalism of resonance driving terms (RDTs). A full quantitative treatment requires the machinery of perturbation theory and Hamiltonian analysis which will be part of the author's thesis but it is sufficient here to show that resonances are excited when the symmetry of a lattice is broken because the exact same mechanism leads to islands. We will only look at first-order geometric (on-momentum) RDTs given by [12]

$$f = \frac{h}{e^{i\Gamma} - 1}, \tag{1}$$

where $\Gamma = 2\pi[(j-k)Q_x + (l-m)Q_y]$ and we have deliberately omitted the indices $j, k, l.m \in \mathbb{N}$ on the variables to prevent the cluttering of notation and the term h for example should really be written as h_{jklm} for on-momentum particles where $j + k + l + m = 3$ to first-order. The driving term h can be found in [2] and can be written in a generic integral form as

$$h = \int_0^L ds\, a(s) e^{i\lambda(s)}, \tag{2}$$

where L is the length of the beamline, λ is related to the phases $\mu_{x,y}$ as $\lambda(s) = (j-k)\mu_x(s) + (l-m)\mu_y(s)$ and [12,13]

$$a(s) = -\frac{i^{l+m} k_2(s)}{j!\,k!\,l!\,m!\,8}\, \beta_x(s)^{(j+k)/2} \beta_y(s)^{(l+m)/2}. \tag{3}$$

The sextupole distribution is k_2 and $\beta_{x,y}$ are the usual Twiss beta-functions. The sums in [2] are recovered by formally replacing $k_2(s)$ with a sum of delta-function kicks. The integral in Eq. (2) is usually evaluated numerically but we will follow the analytical path to gain insight by changing the integration variable from s to λ:

$$h = \int_0^\Gamma d\lambda\, b(\lambda) e^{i\lambda}, \tag{4}$$

where $b = a(ds/d\lambda)$ and the usual relationship $d\mu_{x,y}/ds = 1/\beta_{x,y}$ holds. For the periodic case, we can write $b(\lambda)$ as a Fourier decomposition with period Γ/N where N is the number of superperiods:

$$b_{\text{per}}(\lambda) = \sum_r b_r e^{2\pi i r \lambda N/\Gamma}. \tag{5}$$

Substituting in the integral gives the periodic solution

$$\begin{aligned} h_{\text{per}} &= (e^{i\Gamma} - 1) \sum_r \frac{b_r}{1 + 2\pi r N/\Gamma} \\ &= (e^{i\Gamma} - 1) b_0. \end{aligned} \tag{6}$$

where we have skipped a few straightforward intermediate steps. We immediately see that the denominator in Eq. (1) is cancelled for a periodic lattice. A general perturbation can be written as

$$b_{\text{per}} \to b_{\text{per}} + \Delta b(\lambda), \quad \lambda \to \lambda + \Delta\lambda \tag{7}$$

where $\Delta\lambda = 0$ corresponds to the case where the symmetry is broken with a nonlinear element (sextupole in this case) and there is no beta-beating i.e. $\Delta\beta_{x,y} = 0$. In a robust lattice, h is nearly invariant under the transformation in Eq. (7). Replacing this transformation in Eq. (4) yields

$$h = (e^{i\Gamma} - 1) b_0 + \int_0^\Gamma d\lambda\, [\Delta b(\lambda) e^{i\lambda} + A(\Delta\lambda)] \tag{8}$$

where $A(\Delta\lambda)$ contains powers of $\Delta\lambda$ and by definition $A(\Delta\lambda = 0) = 0$. Let us assume, for simplicity, that $A(\Delta\lambda)$ is small and can be neglected (≈ 0) but $\Delta\beta_{x,y} \neq 0$ and the symmetry is broken at a single location λ_j such that $\Delta b(\lambda) = \Delta b_j \delta(\lambda - \lambda_j)$. The solution for the RDT is therefore

$$f = f_{\text{per}} + \Delta f = b_0 + \frac{\Delta b_j e^{i\lambda_j}}{e^{i\Gamma} - 1}, \tag{9}$$

where the periodic solution for the RDT is $f_{\text{per}} = b_0$. We now see that resonances are already excited in the presence of a single perturbation i.e. the RDT diverges when the resonance condition $\Gamma/2\pi \in \mathbb{N}$ is satisfied and usually, $|\Delta f| \gg |f_{\text{per}}|$ close to a resonance. In this case of a single perturbation, the control parameters are Δb_j and Γ. Δb_j can be used to control for example the size of the islands and Γ their position relative to the main orbit. In the hypothetical case of a linear ring with a single sextupole perturbation i.e. $b(\lambda) = b_j \delta(\lambda - \lambda_j)$, it is clear that the cancellation of numerator and denominator in the RDTs cannot be achieved and resonances are always excited. The situation is expected to be more complicated and less controllable if there is a set of random perturbations at locations $\{\lambda_j\}$ such that $\Delta b(\lambda) = \sum_j \Delta b_j \delta(\lambda - \lambda_j)$ which would correspond to a more realistic lattice.

A FIRST IMPLEMENTATION

As a first step towards establishing TRIBs in the SF lattice, its symmetry was broken by changing the strength k_1 of the last quadrupole of the triplet (matching) section in

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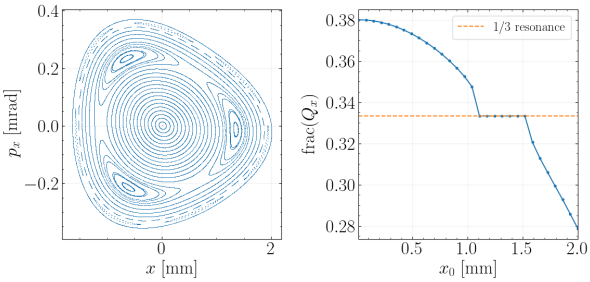
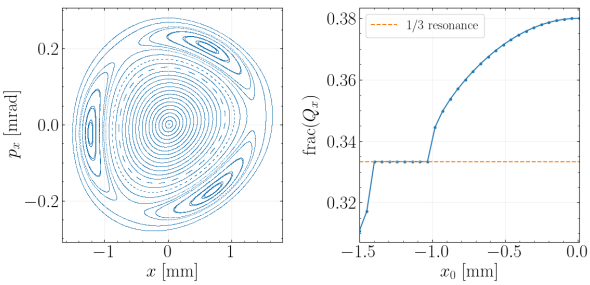


(a) $\Delta k_1/k_1 = -2\%$



(b) $\Delta k_1/k_1 = +2\%$

Figure 3: Horizontal phase space with broken symmetry.

the last superperiod by $\Delta k_1 = \pm 0.2$ m^{-1} corresponding to a fractional change of $\Delta k_1/k_1 = \pm 2\%$. The tracking result is shown in Fig. 3. The working point was kept the same by the other two quadrupoles in the triplet section and the chromaticity is always corrected to $(0, 0)$ using the two families of chromatic sextupoles.

The island structure is now present in the horizontal phase space and the plateau (flat part) in the FFT plot corresponds to the resonant (or island) tune. We can use the width of the plateau Δx_0 as a convenient definition of the island size for our current purposes but a more realistic definition would be the phase-space area enclosed by the outermost particle on the island. A symmetric lattice obviously has $\Delta x_0 = 0$. Although we have showed the Poincaré section at a particular position in the lattice ($s = 0$ in this case), the symplectic condition (or Liouville's theorem) forbids the fixed points and hence the islands from disappearing along the beamline.

The biggest disadvantage of using quadrupoles to establish islands is the modification of the linear optics as can be seen in Fig. 4 and in a real lattice, additional elements are required to minimize this beta-beat; the correction is never perfect. The reason why a quadrupole was used here was to keep the lattice in its most basic (earliest design) version i.e. without multipoles in the non-dispersive regions to optimize its nonlinear dynamics. The length of any element can also be used as a symmetry breaking knob for $\Delta\beta_{x,y}$ but this case would be very unrealistic and unachievable.

It can be demonstrated using normal form analysis [14] that if $\mathcal{M}$ is the one-turn map of the ring, the closed orbit for a $p/3$ island is mapped onto itself by $\mathcal{M}^3$ instead of $\mathcal{M}$. In a Poincaré section from tracking, this means that the closed orbit "jumps" around and comes back to its original position

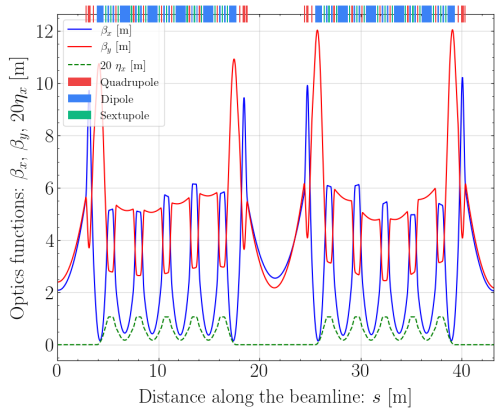


Figure 4: Beta-beat for $\Delta k_1/k_1 = -2\%$.

after three turns. In Fig. 5 we show, via simulations, the evolution of the closed orbit X_{co} (main and island) along the beamline and it can be seen that the island orbit closes on itself after three turns compared to one turn for the main orbit.

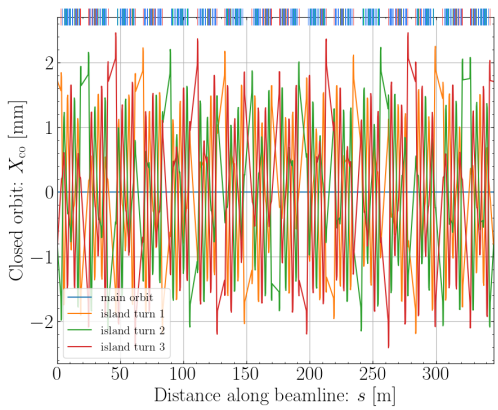


Figure 5: Closed orbits for $\Delta k_1/k_1 = -2\%$.

SUMMARY AND OUTLOOK

We showed that for the SF design lattice, the symmetry of the lattice had to be broken to establish islands in simulations where we used a quadrupole in the matching cell as the symmetry breaking knob. Although the material presented here is somewhat academic and still work in progress, more realistic investigations are planned. For example, multipoles will possibly be added in the non-dispersive region as knobs for islands because they do not alter the linear optics and secondly to control the tune-shift with amplitude. Engineering tolerances also need to be added to identify the "strongest" knobs which can possibly control the islands (size, location, ...) to some extent. To sum up, the tentative goal is to have at some point a reliable and robust implementation of TRIBs by increasing the level of complexity towards a more realistic lattice.

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