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STUDIES OF THE VERTICAL EXCURSION FIXED FIELD ALTERNATING GRADIENT ACCELERATOR

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Abstract

The Vertical Excursion Fixed Field Alternating Gradient Accelerator (VFFA) concept offers a number of advantages over existing accelerator archetypes, as discussed in previous works [1, 2]. However, the VFFA has nonplanar orbits by design and unavoidable transverse coupling. Hence, current understanding of the dynamics of this machine is limited; this paper presents some in-depth study of its behaviour using a combination of analytical and numerical techniques.

INTRODUCTION

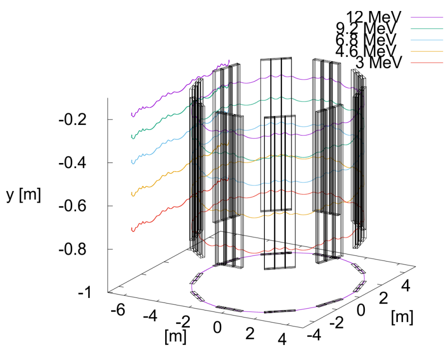

Figure 1: Closed orbits in a 10 cell VFFA FDF triplet ring at a variety of energies, showing how orbits increase in vertical height as energy increases.

In the VFFA, successive higher-energy orbits appear as vertically translated copies of lower-energy orbits (Figure 1).To ensure constant tune, magnetic fields scale as

$$\vec{B}(X,Y,Z) = \vec{B}(X,Y_0,Z)e^{m(Y-Y_0)}, \tag{1}$$

in which X, Y, and Z represent the transverse horizontal and vertical axes and the longitudinal direction respectively. m is the VFFA field index, with units m^{-1}, and defines the spatial separation of successive orbits.

Substituting this scaling criterion into Maxwell's equations, the following polynomial expansion for the magnetic fields associated with a VFFA magnet can be derived:

$$\begin{aligned} B_X &= B_0 e^{mY} \sum \frac{n+1}{m} f_{n+1} X^n \\ B_Y &= B_0 e^{mY} \sum f_n X^n \\ B_Z &= B_0 e^{mY} \sum \frac{1}{m}\frac{df_n}{dZ} X^n \end{aligned} \tag{2}$$

$$\begin{aligned} f_0 &= f(Z), f_1 = 0 \\ f_{n+2} &= -\frac{1}{(n+1)(n+2)}\left(\frac{d^2 f_n(Z)}{dZ^2} + m^2 f_n(Z)\right), \end{aligned} \tag{3}$$

in which $f(Z)$ denotes the distribution of the field along the magnet's longitudinal axis.

Several properties with notable implications for the dynamics of the VFFA can be observed from the above field expansion. Firstly, the horizontal B-field is nonzero away from the midplane, and where $f(Z)$ is increasing or decreasing (i.e. in the fringe field regions) there will exist longitudinal field components. This means, in practice, that any VFFA will have an orbit with significant vertical deviation from a horizontal plane, and that its optics are fundamentally coupled in a nontrivial manner.

An ideal model of this machine would be completely analytic, able to determine the machine's properties purely from the input parameters of the lattice. This would lead to rapid design and optimisation processes across the parameter space, and reduce the need for intensive simulations. However, the complexities discussed above render the development of an analytic model challenging, and as such we turn to numerical studies to inform the construction of an analytic approach.

There are two key elements in understanding the behaviour of any accelerator: the determination of the closed orbit; and the optics associated with the magnetic field about the closed orbit. We begin with a numerical study of the multipole fields about a pre-determined closed orbit.

NUMERICAL STUDIES

First, the closed orbit of a cell is determined using an existing tracking code such as FIXFIELD [3]. This tool is also used to compute a pair of tunes in the decoupled u and v planes of motion. The closed orbit evaluated from the tracking code then forms the basis for a technique we term harmonic analysis: at each point along the closed orbit, magnetic fields are evaluated around a circle of radius dr in a plane perpendicular to the orbit. The field at the centre of the circle (i.e. on the closed orbit) is subtracted from the fields on its circumference and divided by the radius to calculate the relative field gradient. These fields are then decomposed into horizontal and radial components, and the Fourier transform of the radial field gradient is taken. By applying an appropriate normalisation, this then allows us to obtain the multipole coefficients of a decomposition of the focussing fields about the closed orbit. The longitudinal field with respect to the closed orbit is also measured, in order to evaluate any solenoid-type effects.

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Multipole coefficients generated in this way can then be used to set transfer matrix elements in a simple kick-drift integration scheme, which can be used to reconstruct the optics of the system and study the relative impact of each component of the multipole decomposition.

Muon Accelerator Lattice

As a simple test case, a straight (i.e. zero net bending per cell) FODO lattice was constructed based on the specifications for a muon accelerator lattice for a muon collider (Table 1) [4].

Table 1: Muon Accelerator Lattice Parameters

Parameter	**Baseline Value**
Cell Length	35 m
Magnet Length	12 m
FD Ratio ($\frac{B_{0d}}{B_{0f}}$)	1
Fringe Field Extent	0.2 m
m-value	4m
Horizontal Magnet Offset	0.37 m

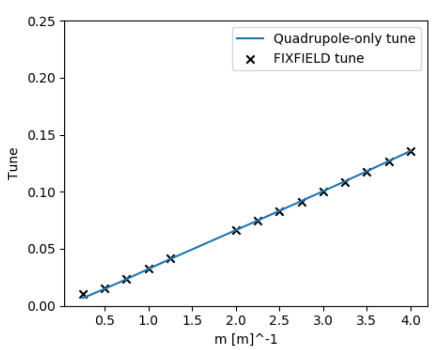

Figure 2: m-value dependence of the decoupled tunes in both a quadrupole-only model and full numerical tracking.

This represents an ideal test case for the simplest possible formulation of VFFA optics. Long magnets in comparison to short fringe fields means that the properties of the system should be determined primarily by the magnet body behaviour; a large local radius of curvature reduces the impacts of geometric effects (weak focussing), and also means that the distance of the closed orbit from the magnet midplane remains approximately constant. Figure 2 shows the evolution of the tune using a quadrupole-only computation with coefficients set based on the harmonic analysis, compared to the tune from the FIXFIELD simulation. This shows that for all variations of this FODO geometry, the tune can be well-predicted using simple (skew) quadrupole optics, and the behaviour of the lattice is indeed dominated by the magnet body dynamics in this regime.

Small VFFA Lattice Studies

Current proposals for VFFA machines [2] are based around smaller cells, which are likely to have significantly

Table 2: Small VFFA Lattice Parameters

Parameter	**Baseline Value**	
	FODO	Triplet
Cell Length	2.8 m	2.8 m
Magnet Length	0.5 m	0.5 m
Fringe Field Extent	0.15 m	0.15 m
FD Ratio ($\frac{B_{0d}}{B_{0f}}$)	1	2
m-value	4/m	4/m
Horizontal Magnet Offset	±0.02 m	±0.02 m
Drift Length Inside Triplet		0.03 m

larger degrees of local bending (increasing the magnitude of geometric effects compared to the previous case), as well as more dominant fringe field effects (i.e. the fringe fields are of comparable length to the magnet lengths). In order to study these contributions, two lattices based on a smaller VFFA ring have been developed. In this case, each cell has the same length, and zero net bending angle per cell, but one is a FODO lattice (similar to the muon accelerator ring) whilst the other is an FDF triplet lattice (see Table 2).

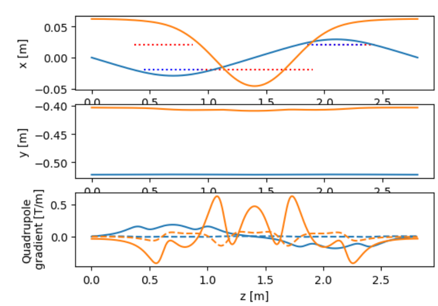

Figure 3: x (horizontal) and y (vertical) trajectories of the closed orbits for the small FODO and Triplet lattices, along with the normal (dotted line) and skew (solid line) quadrupole gradients in each case. The FODO lattice is represented by the blue lines, whilst the triplet lattice results are displayed with orange lines. The vertical midplanes of the magnets in the FODO and Triplet lattices are displayed in the first subplot with dashed blue and red lines respectively.

Figure 3 displays the closed orbits and quadrupole decompositions for each of the above cases.

The u and v tunes calculated for the FODO lattice via quadrupole-only kick-drift integration are (0.0907, 0.0909). This compares to (0.0808, 0.0746) from the full numerical integration - a relatively small discrepancy. However, for the triplet lattice, the tunes calculated using the quadrupole-only method are (0.162, 0.117) - which does not agree with the values from FIXFIELD's tune calculation at (0.245, 0.182).

This implies the existence of other significant contributions to the tune that are apparent in the triplet lattice, but much less significant for the equivalent FODO lattice. Figure 4a shows the longitudinal field components in the two lattices. It can be seen that the longitudinal fields are substan-

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tially larger in magnitude for the triplet lattice, particularly where the fringes of the F and D magnets overlap in the triplet. The presence of stronger longitudinal fields and solenoid-type behaviour in the triplet lattice may to some extent explain the discrepancy in tune when compared to the computational model, and this must be investigated further.

Similarly, Figure 4b shows the evolution of dipole fields through the cell for both the FODO and triplet lattices. Once again, the magnitude of fields here is greater in the triplet case than the FODO case, and we would expect a larger influence from geometric effects (weak focussing) in the triplet. As the VFFA is a strong-focussing machine, it is expected that weak focussing effects should have only a small impact on the tune, but this is another possible explanation for the observed discrepancy.

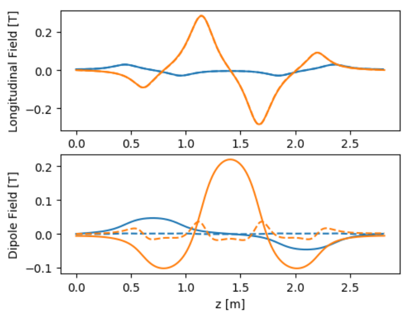

Figure 4: Longitudinal and dipole field contributions along the closed orbit for the FODO (blue) and Triplet (orange) lattices. The vertical B-field is denoted by the solid line in the second figure, whilst the horizontal is represented with the dashed line.

ANALYTIC RESULTS

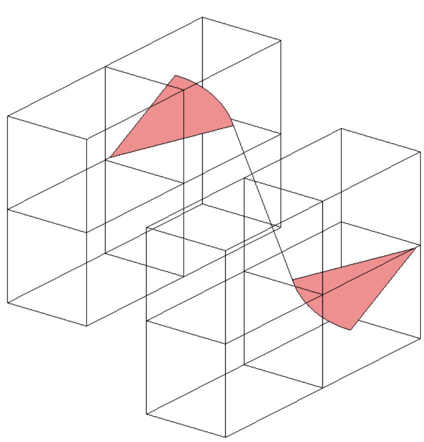

Figure 5: A diagram of the closed orbit through a half-cell in a straight FODO lattice. The red highlighted region represents the plane of curvature within the magnets.

The studies of the muon collider lattice show that the dynamics of a simple VFFA can be well predicted with quadrupole-order optics, provided appropriate coefficients are known. It is then necessary to develop an analytic method to determine the parameters of the closed orbit and thus work out the values of these coefficients. By examining the geometry of the FODO cell (Figure 5), it is possible to derive a set of simultaneous equations that parametrise the geometry of the closed orbit in terms of the input parameters x_m (horizontal offset of magnet midplane from the x-axis), L_f and L_d (lengths of F and D magnets respectively), L_s (drift length between magnets), and the FD-ratio (ratio of B field in F magnet to B field in D-magnet at constant height). These equations can be solved for ρ_F and ρ_D (the radii of curvature in each magnet) as well as x_{0f} and x_{0d} (the horizontal distance from closed orbit to midplane). Using the fact that ρ is equal to the dipole field strength B_0 over the beam rigidity, a magnet body Hamiltonian can be derived from the VFFA field expansion (Equation 2) and the standard Frenet-Serret Hamiltonian, (neglecting higher-order and dipole order terms):

$$H = \frac{P_x^2}{2} + \frac{P_y^2}{2} + \frac{m}{\rho}(1 - \frac{m^2 x_0^2}{2})xy + \frac{m^2 x_0}{2\rho}x^2 - \frac{m^2 x_0}{2\rho}y^2 + \frac{m^4 x_0^3}{12\rho}y^2., \quad (4)$$

Constructing magnet transfer matrices from this Hamiltonian and substituting in the parameters determined from the geometry, a cell transfer matrix is derived and used to compute the tune. The performance of tune predictions from this fully analytic model is compared to the results from numerical simulation in Figure 6.

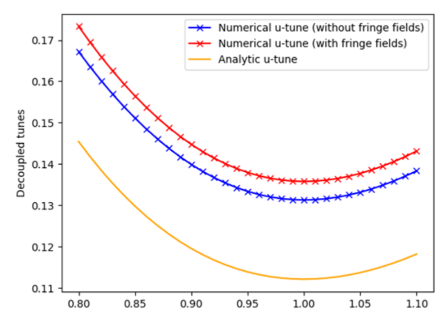

Figure 6: Tune from a fully analytic model compared to tune from numerical simulations as the FD ratio is varied.

CONCLUSION

It is now possible to predict the tune behaviour of a simplest-case FODO VFFA using a completely analytic model. This model must now be refined and adapted in two regards: the analytic determination of the closed orbit needs to be modified to account for more complex lattices with nonzero bending angle; and the optical modelling must be enhanced to include effects from solenoid-like fields and geometric focussing as identified using the harmonic analysis method.

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