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AN N-BPM MOMENTUM RECONSTRUCTION FOR LINEAR TRANSVERSE COUPLING MEASUREMENTS IN LHC AND HL-LHC

A. Wegscheider and R. Tomás, CERN, Geneva, Switzerland

Abstract

The measurement and control of linear transverse coupling is important for the operation of an accelerator. The calculation of the linear transverse coupling resonance driving terms (RDTs) f_{1001} and f_{1010} relies on the complex spectrum of the turn-by-turn motion. To obtain the complex signal, a reconstruction of the particle motion is needed. For this purpose, the signal of a second BPM with a suitable phase shift is usually used. In this work, we explore the possibility of including more BPMs in the reconstruction of the transverse momentum, which could reduce the effects of statistical errors and systematic uncertainties. This, in turn, could improve the precision and accuracy of the RDTs, which could be of great benefit for locations where an exact knowledge of the transverse coupling or other RDTs is important. We present the development of a new method to reconstruct the particle's momentum that uses a statistical analysis of several nearby BPMs. The improved precision is demonstrated via simulations of LHC and HL-LHC lattices.

COUPLING FROM TURN-BY-TURN DATA

Calculating Coupling Resonance Driving Terms

The measurement and control of linear transverse coupling is important for the operation of an accelerator. In the LHC, the target is usually to completely eliminate coupling, which makes a high precision of the measurement crucial. Coupling measurements were steadily improved over the first two operational runs [1–6] and advanced coupling measurement techniques were proposed in recent years [7–9].

The coupling RDTs

$$f_{1001} = |f_{1001}|e^{iq_{1001}} \quad \text{and} \tag{1}$$
$$f_{1010} = |f_{1010}|e^{iq_{1010}} \quad , \tag{2}$$

can be calculated from the turn-by-turn spectrum [10] by

$$|f_{1001}| = \frac{1}{2}\sqrt{\left|H^{+}_{0,1}V^{+}_{1,0}\right|}\,, \tag{3}$$
$$|f_{1010}| = \frac{1}{2}\sqrt{\left|H^{+}_{0,-1}V^{+}_{-1,0}\right|} \quad , \tag{4}$$

where $H^{+}_{n_x,n_y}$ and $V^{+}_{n_x,n_y}$ are the complex horizontal and vertical spectral lines with frequencies $n_xQ_x + n_yQ_y$. Q_x and Q_y are the horizontal and vertical tunes. $H^{+}_{n_x,n_y}$ and $V^{+}_{n_x,n_y}$ are normalised by the amplitudes of the main lines with frequencies Q_x for H and Q_y for V, such that $H^{+}_{1,0} = V^{+}_{0,1} = 1$. The phase of the RDTs can be retrieved – from either the horizontal or vertical signal – by

$$\begin{aligned} q_{1001} &= -\arg(H^{+}_{0,1}) - \varphi^{\mathrm{m}}_{x,ab} + \frac{\pi}{2} \\ &= \arg(V^{+}_{1,0}) + \varphi^{\mathrm{m}}_{y,ab} - \frac{\pi}{2}\,, \end{aligned} \tag{5}$$
$$\begin{aligned} q_{1010} &= -\arg(H^{+}_{0,-1}) + \varphi^{\mathrm{m}}_{x,ab} + \frac{\pi}{2} \\ &= -\arg(V^{+}_{-1,0}) + \varphi^{\mathrm{m}}_{y,ab} + \frac{\pi}{2} \quad , \end{aligned} \tag{6}$$

where the phase advance φ_{ab} between two locations a and b is defined as $\varphi_{ab} = \varphi(s_b) - \varphi(s_a)$ and s_a denotes the longitudinal position of location a.

Unfortunately we cannot measure the particle's relative transverse momentum required to construct the complex signal. A *reconstruction* of the real spectrum using the position data at two nearby BPMs is possible, using the following equations:

$$H^{+}_{n_x,n_y} = \frac{1}{2}\left[(1-\tan\Delta)H^{a}_{n_x,n_y} - \frac{i}{\cos\Delta}H^{b}_{n_x,n_y}\right]\,, \tag{7}$$
$$V^{+}_{n_x,n_y} = \frac{1}{2}\left[(1-\tan\Delta)V^{a}_{n_x,n_y} - \frac{i}{\cos\Delta}V^{b}_{n_x,n_y}\right]\,, \tag{8}$$

where $H^{a}_{n_x,n_y}$ is the real horizontal spectral line at position s_a, analogously for the vertical signal and Δ is the deviation of the phase advance from $\pi/2$:

$$\Delta = \varphi_{ab} - \frac{\pi}{2} \quad . \tag{9}$$

Equations (7) and (8) assume that there are no additional coupling sources in between the positions a and b [11].

Phase Measurement Errors

Equations (7) and (8) are sensitive to phase measurement errors which get enhanced when the model phase advance is near a root of the cos term in the denominator. To avoid exploding errors, the conventional method is a careful selection of suitable BPM pairs.

The current implementation of the calculation of coupling in our python tool set [12, 13] features two different modes of selecting BPM pairs. The first one pairs each BPM with a second BPM j positions downstream. In the LHC arcs the phase advance between one BPM and the one 2 positions downstream is close to $\pi/2$, which is optimal for the momentum reconstruction.

The second method pairs each BPM with a second BPM downstream which has a phase advance of approximately $\pi/2$ with respect to the first one. This selective method guarantees that the pairing uses optimal phase advances. Since Eqs. (7) and (8) assume no coupling sources in between positions a and b, skipping too many BPMs increases the chances of picking up coupling errors in between, so a limit in the maximum amount of skipped BPMs has to be set.

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N-BPM RECONSTRUCTION

The new approach, presented in this work, uses the measurements at several BPM positions to improve statistics and precision of the momentum reconstruction, similar to the method used to measure a more precise β function [14, 15].

Similar to the calculation of the β function, we compute the RDTs for different BPM pairs and weight them according to their statistical errors,

$$|f_{1001}|(s_a) = \sum_l g_l |f_{1001}|_l(s_a) \tag{10}$$

$$|f_{1010}|(s_a) = \sum_l g_l |f_{1001}|_l(s_a) \quad , \tag{11}$$

$$\text{with weights} \quad g_l = \frac{\sum_k V_{lk}^{-1}}{\sum_{i,j} V_{ij}^{-1}} \quad , \tag{12}$$

where $\mathbf{V}$ denotes the covariance matrix for the variable (f_{1001} or f_{1010}).

The covariance matrix can be calculated by

$$\mathbf{V} = \mathbf{TMT}^{\dagger} \tag{13}$$

with $\mathbf{M} = \mathrm{diag}(\sigma_1^2, \ldots \sigma_n^2)$ a diagonal matrix consisting of the variances of the observables x_i, $\mathbf{T}$ the Jacobian

$$T_{ij} = \frac{\partial f_i}{\partial x_j} \tag{14}$$

and $\mathbf{T}^{\dagger}$ denoting the conjugate transpose of $\mathbf{T}$. f_i stands for either RDT at position i.

The task is to collect different error sources and incorporate the error propagation in the covariance matrix. Multiple sources are relevant: statistical errors, such as the phase and amplitude measurement error from turn-by-turn data and systematic errors such as quadrupole and BPM tilts [16]. A consideration of the systematic uncertainties is beyond the scope of this work and we will concentrate on phase measurement errors only, leaving the remaining error sources (amplitude measurement errors and systematic errors) for future studies.

STATISTICAL UNCERTAINTIES

The measurement of the phase advances φ_{ab} suffers from statistical measurement errors $\sigma_{\varphi_{ab}}$. The Jacobian for this part can be obtained by taking the partial derivative of Eq. (4)

$$\begin{aligned} T_{ij}^{1001} &= \frac{\partial |f_{1001}|_i}{\partial \varphi_j} \\ &= \frac{\mathrm{sgn}\left(H_{0,1}^{+} V_{1,0}^{+}\right)}{\sqrt{\left|H_{0,1}^{+} V_{1,0}^{+}\right|}} \left(H_{0,1}^{\prime +} V_{1,0}^{+} + H_{0,1}^{+} V_{1,0}^{\prime +}\right) \end{aligned} \tag{15}$$

and, for f_{1010},

$$\begin{aligned} T_{ij}^{1010} &= \frac{\partial |f_{1010}|_i}{\partial \varphi_j} \\ &= \frac{\mathrm{sgn}\left(H_{0,-1}^{+} V_{-1,0}^{+}\right)}{\sqrt{\left|H_{0,-1}^{+} V_{-1,0}^{+}\right|}} \left(H_{0,-1}^{\prime +} V_{-1,0}^{+} + H_{0,-1}^{+} V_{-1,0}^{\prime +}\right) \quad . \end{aligned} \tag{16}$$

The prime in $H_{n_x,n_y}^{\prime +}$ and $V_{n_x,n_y}^{\prime +}$ denotes derivation with respect to φ_{ab}:

$$H_{n_x,n_y}^{\prime +} = \frac{1}{2}\left[H_{n_x,n_y}^{a} - \frac{1}{\cos^2\Delta}\left(H_{n_x,n_y}^{a} + i\sin\Delta H_{n_x,n_y}^{b}\right)\right] , \tag{17}$$

$$V_{n_x,n_y}^{\prime +} = \frac{1}{2}\left[V_{n_x,n_y}^{a} - \frac{1}{\cos^2\Delta}\left(V_{n_x,n_y}^{a} + i\sin\Delta V_{n_x,n_y}^{b}\right)\right] . \tag{18}$$

Since we consider only one error variable (φ), the covariance matrix $\mathbf{V}$ collapses to a single row:

$$\mathbf{V} = \left(\frac{\partial |f|}{\partial \varphi_1} \ldots \frac{\partial |f|}{\partial \varphi_N}\right) \quad , \tag{19}$$

for either f. The weighted average reads

$$|f| = \frac{1}{\sum_{i=0}^{N} g_i} \sum_{i=0}^{N} g_i |f_i| \tag{20}$$

with the weights being

$$g_i^{1001} = \left(\frac{\partial f_{1001}}{\partial \varphi_i}\sigma_{\varphi_i}\right)^2 \quad , \tag{21}$$

$$g_i^{1010} = \left(\frac{\partial f_{1010}}{\partial \varphi_i}\sigma_{\varphi_i}\right)^2 \quad . \tag{22}$$

The measurement errors σ_{φ_i} are the standard deviations of the phase measurements from turn-by-turn data.

VERIFICATION USING DIFFERENT LATTICES

LHC Injection Optics

Since a suppression of coupling has to be ensured at each optics configuration, the first coupling measurement typically takes place at injection. Therefore we study the performance of coupling measurements at this configuration with typical noise levels. The LHC BPMs have a resolution of typically 100 µm [17, 18]. The singular value decomposition cleaning applied by our harmonic analysis tools would clean an artificially introduced Gaussian noise distribution too well. Therefore we add a higher noise to the simulation data and match it with the expected output [19].

We introduce one random coupling source per arc, with a standard deviation of $\sigma_K = 0.5 \times 10^{-3}\,\mathrm{m}^{-1}$.

Figure 1 shows the measured coupling from simulations, calculated by the following three methods: The blue curve shows the conventional method which is skipping one BPM, called `skip 1`, to obtain approximately $\pi/2$ phase advances in the arc. This method performs badly around the IRs where such a phase advance is not assured. The orange curve shows the method which performs a careful selection of BPMs with $\pi/2$ phase advance, risking to pick up errors when the interval becomes large. In green the new N-BPM method is shown. It slightly outperforms the $\pi/2$ method and is also considerably better than the `skip 1` method.

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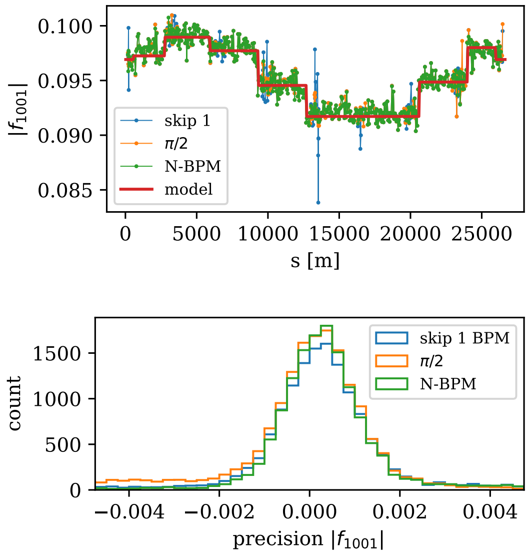

Figure 1: Comparison of the precision of the conventional methods and of the new N-BPM method for a lattice with a random coupling distribution. TOP: measured / model values of $|f_{1001}|$. For the `skip=1` method peaks at the IPs are clearly visible. BOTTOM: the distribution of precision across several measurements. Higher peak and narrower distribution means better overall precision.

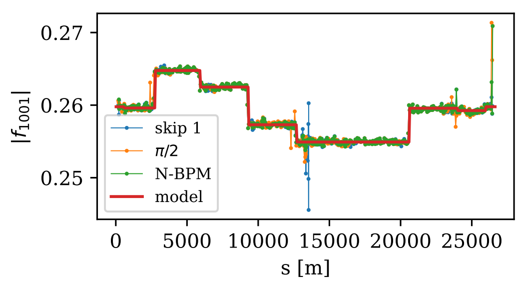

Figure 2: The same comparison as Fig. 1 for squeezed LHC optics at $\beta^* = 30$ cm.

LHC Collision Optics

Tightly controlling the accelerator during the physics run is crucial for its performance and machine safety. This is, in case of the LHC, the optics at collision with $\beta^* = 30$ cm. The noise levels had to be adjusted to reflect actual measurement values. Apart from that, the simulation setup remains unchanged, while a random coupling error is introduced per arc. A comparison of the measurement quality can be found in Figure 2. The `skip 1` method suffers from increasingly strong outliers because of the smaller phase advances around the colliding IPs.

HL-LHC Collision Optics

The last optics configuration that we consider in this work is the HL-LHC optics [20, 21] at $\beta^* = 20$ cm. This configuration shows a similar picture to the LHC case at collision,

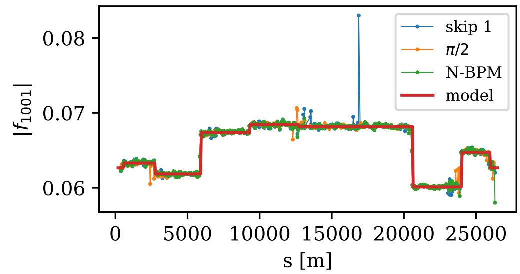

Figure 3: The same comparison as Fig. 1 for HL-LHC collision optics at $\beta^* = 20$ cm.

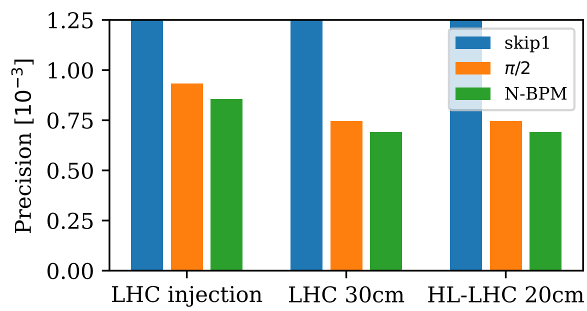

Figure 4: A comparison of root mean square precision of the three methods and all three lattice configurations. The rms of the `skip 1` method is in all three configurations orders of magnitude higher than the other methods because of outliers in the IRs.

as can be seen in Figure 3. Figure 4 shows a comparison of the root mean square precision of all three methods for each of the three lattice configurations. In every case, the new N-BPM method slightly outperforms the $\pi/2$ method. The outliers of the `skip 1` method blow up the rms values by several orders of magnitude.

CONCLUSION AND OUTLOOK

This work presents a new method to calculate linear transverse coupling in circular accelerators that takes into account statistical errors in a refined reconstruction of the particles' momentum. This statistical approach improves the precision of the coupling calculation in a way that is independent of the actual lattice configuration (in particular the phase advance between the BPMs). Systematic uncertainties have not been considered in this work. Introducing them into the picture will improve the performance of the N-BPM method even further.

In a future work one can explore the possibility to include amplitude measurement errors as well as systematic errors in the error propagation, such as quadrupole or BPM tilts.

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REFERENCES

[1] T. Persson *et al.*, "Automatic Correction of Betatron Coupling in the LHC using Injection Oscillations," in *Proc. IPAC'13*, Shanghai, China, May 2013, pp. 1979–1981, https://jacow.org/IPAC2013/papers/TUPWO049.pdf

[2] T. Persson and R. Tomás, "Improved control of the betatron coupling in the Large Hadron Collider," *Phys. Rev. ST Accel. Beams*, vol. 17, no. 5, p. 051 004, 2014, doi:10.1103/PhysRevSTAB.17.051004

[3] T. Persson *et al.*, "Experience with DOROS BPMs for Coupling Measurement and Correction," in *Proc. IPAC'16*, Busan, Korea, May 2016, pp. 303–305, doi:10.18429/JACoW-IPAC2016-MOPMR029

[4] T. Persson *et al.*, "Transverse Coupling Measurements With High Intensity Beams Using Driven Oscillations," in *Proc. IPAC'18*, Vancouver, Canada, Apr.-May 2018, pp. 208–211, doi:10.18429/JACoW-IPAC2018-MOPMF047

[5] G. H. Hemelsoet *et al.*, "Online Coupling Measurement and Correction Throughout the LHC Cycle," in *Proc. ICALEPCS'17*, Barcelona, Spain, Oct. 2017, pp. 686–691, doi:10.18429/JACoW-ICALEPCS2017-TUPHA119

[6] R. Tomás, M. Aiba, A. Franchi, and U. Iriso, "Review of linear optics measurement and correction for charged particle accelerators," *Phys. Rev. Accel. Beams*, vol. 20, p. 054 801, 5 2017, doi:10.1103/PhysRevAccelBeams.20.054801

[7] F. Soubelet, O. Apsimon, T. H. B. Persson, R. T. García, and C. P. Welsch, "Prospect for Interaction Region Local Coupling Correction in the LHC Run 3," pp. 61–64, doi:10.18429/JACoW-IPAC2021-MOPAB007

[8] E. J. Høydalsvik and T. H. B. Persson, "Reaching the Sub Per Mil Level Coupling Corrections in the LHC," pp. 3752–3755, doi:10.18429/JACoW-IPAC2021-THPAB001

[9] A. Wegscheider and R. T. García, "Forced Coupling Resonance Driving Terms," in *Proc. IPAC'21*, Campinas, Brazil, May 2021, pp. 3646–3649, doi:10.18429/JACoW-IPAC2021-WEPAB400

[10] OMC team. "Chapter about coupling on the OMC website." (2022), https://pylhc.github.io/measurements/physics/coupling/

[11] R. Tomás, M. Bai, R. Calaga, W. Fischer, A. Franchi, and G. Rumolo, "Measurement of global and local resonance terms," *Phys. Rev. ST Accel. Beams*, vol. 8, p. 024 001, 2 2005, doi:10.1103/PhysRevSTAB.8.024001

[12] OMC-Team *et al.*, *OMC3*, version 0.2.7, 2022, doi:10.5281/zenodo.6503905

[13] OMC-Team *et al.*, *BetaBeat.src*, https://github.com/pylhc/Beta-Beat.src

[14] A. Langner and R. Tomás, "Optics measurement algorithms and error analysis for the proton energy frontier," *Phys. Rev. ST Accel. Beams*, vol. 18, no. 3, p. 031 002, 2015, doi:10.1103/PhysRevSTAB.18.031002

[15] A. Wegscheider, A. Langner, R. Tomás, and A. Franchi, "Analytical N beam position monitor method," *Phys. Rev. Accel. Beams*, vol. 20, no. 11, p. 111 002, 2017, doi:10.1103/PhysRevAccelBeams.20.111002

[16] L. van Riesen-Haupt and R. T. García, "The Impact of Beam Position Monitor Tilts on Coupling Measurements," in *Proc. IPAC'21*, Campinas, Brazil, May 2021, pp. 1342–1345, doi:10.18429/JACoW-IPAC2021-TUPAB006

[17] R. Tomás, *BPM noise in different accelerators*, 2021, https://indico.cern.ch/event/1038220/contributions/4375967/attachments/2257593/3831105/BPM%20noise%20for%20various%20accelerators.pdf

[18] L. Malina and R. Tomás, *BPMs in turn-by-turn mode: LHC experience and HL-LHC requirements*, 2019, https://indico.cern.ch/event/840137/contributions/3524620/attachments/1901254/3138571/20190903_BPM_performance_WP2.pdf

[19] F. P. G. Soubelet and R. Tomas Garcia, "Measured Phase Error Dependency on BPM Type and Location in the LHC," 2021, https://cds.cern.ch/record/2751179

[20] X. Buffat *et al.*, "Optics Measurement and Correction Strategies for HL-LHC," CERN, Tech. Rep., 2022, https://cds.cern.ch/record/2808650

[21] R. Tomas Garcia *et al.*, "HL-LHC Run 4 proton operational scenario," CERN, Tech. Rep., 2022, https://cds.cern.ch/record/2803611